

# Introduction

Please allow us to introduce you to Ashley Norris, Maria Rios, Benjamin Wittrock, and Li Shuzhen. These four mathematics teachers set out each day to deliberately, intentionally, and purposefully impact the learning of their students. They recognize these important elements:

- They have the capacity to select and implement various teaching and learning strategies that enhance their students' learning in mathematics.
- The decisions they make about their teaching have an impact on student learning.
- Every student can learn mathematics, and they need to take responsibility to teach *all* learners.
- They must continuously question and monitor the impact of their teaching on student learning (adapted from Hattie & Zierer, 2018).

Through the videos accompanying this book, you will meet additional secondary mathematics teachers and the instructional leaders who support them in their teaching. Collectively, the **mindframes** of these teachers—or their ways of thinking about teaching and learning—lead to action in their mathematics classrooms, and their actions lead to outcomes in student learning. This is where we begin our journey through *Teaching Mathematics in the Visible Learning Classroom*.

Visible Learning occurs when teachers *see* learning through the eyes of their students and students *see* themselves as their own teachers. How do teachers of mathematics see relations, functions, equations, geometric proofs, trigonometric identities, and logarithmic functions through the eyes of their students? In turn, how do teachers develop

**Mindframes** are ways of thinking about teaching and learning. Teachers who possess certain ways of thinking have major impacts on student learning.

assessment-capable visible learners—students who see themselves as their own teachers—in the study of numbers, operations, and relationships? Conceptualizing, implementing, and sustaining Visible Learning in the secondary mathematics classroom by identifying *what works best* and *what works best when* is exactly what we set out to do in this book.

Mathematics learning involves the balance of conceptual understanding, procedural knowledge, and the application of concepts and thinking skills to a variety of mathematical contexts. By *balance*, we mean that no one dimension of mathematics learning is more important than the other two. Conceptual understanding, procedural knowledge, and the application of concepts and thinking skills are each essential aspects of learning mathematics. Mathematics classrooms where *teachers see learning through the eyes of their learners and learners see themselves as their own teachers* result from specific, intentional, and purposeful decisions about each dimension of mathematics instruction critical for student growth and achievement. This book explores each of these components in secondary mathematics teaching and learning through the lens of *what works best* in student learning at the surface, deep, and transfer phases. We are not suggesting that teachers implement procedural knowledge, conceptual understanding, and application in isolation, but through a series of linked learning experiences and challenging mathematical tasks that result in students engaging in both mathematical content and practices or processes.

**Our Learning Intention:** To understand what works best in the secondary mathematics classroom.

## What Works Best

Identifying what works best draws from the key findings from *Visible Learning* (Hattie, 2009) and also guides the classrooms described in this book. One of those key findings is that *there is no one way to teach mathematics or one best instructional strategy that works in all situations for all students*, but there is compelling evidence for certain strategies and approaches to have a greater likelihood of helping students reach their learning goals. In this book, we use the effect size information that John Hattie has collected and analyzed over many years to inform how we transform the findings from the Visible Learning research into learning experiences and challenging mathematical tasks that are most likely to have the strongest influence on student learning.

For readers less familiar with Visible Learning, we would like to take a moment to review what we mean by *what works best*. The Visible Learning database is composed of over 1,800 meta-analyses of studies that include over 80,000 studies and 300 million students. Some have argued that it is the largest educational research database amassed to date. To make sense of so much data, John Hattie focused his work on meta-analyses. A **meta-analysis** is a statistical tool for combining findings from different studies with the goal of identifying patterns that can inform practice. In other words, a meta-analysis is a study of studies. The mathematical tool that aggregates the information is an effect size and can be represented by Cohen's *d*. An **effect size** is the magnitude, or relative size, of a given effect. Effect size information helps readers understand not only that something does or does not have an influence on learning, but the relative impact of that influence.

For example, imagine a hypothetical study in which learning mathematics while walking on a treadmill results in relatively higher mathematics scores. Schools and classrooms around the country might devote large monetary resources to buying treadmills for mathematics classrooms. However, let's say the results of this hypothetical study indicate that the "treadmill effect" had an effect size of 0.03 in mathematics achievement when compared to those students that did not walk on a treadmill, an effect size pretty close to zero. Furthermore, the large number of students participating in the study made it almost certain there would be a difference in the two groups of students (those using a treadmill vs. those not using a treadmill). As an administrator or teacher, would you still advocate for spending a large amount of your district or school budget on treadmills? How confident would you be in the impact or influence of your decision on mathematics achievement in your district or school?

This is where an effect size of 0.03 for the "treadmill effect" is helpful. Understanding the effect size helps us know how powerful a given influence is in changing achievement—in other words, the impact for the effort or return on the investment. The effect size does not just help us understand what works, but what works *best*. With the increased frequency and intensity of mathematics initiatives, programs, and packaged curricula, deciphering where to best invest resources and time to achieve the greatest learning outcomes for all students is challenging and frustrating. For example, some programs or packaged curricula are

A **meta-analysis** is a statistical tool for combining findings from different studies with the goal of identifying patterns that can inform practice.

**Effect size** represents the magnitude of the impact that a given approach has on learning.



### Video 1 What Is Visible Learning for Mathematics?

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand. Videos can also be accessed at <https://resources.corwin.com/vlmathematics-9-12>

hard to implement and have very little impact on student learning. Some programs and packaged curricula are easy to implement and still have limited influence on student growth and achievement in mathematics. Teaching mathematics in the Visible Learning classroom involves searching for those things that have the greatest impact and produce the greatest gains in learning, some of which will be harder to implement and some of which will be easier to implement.

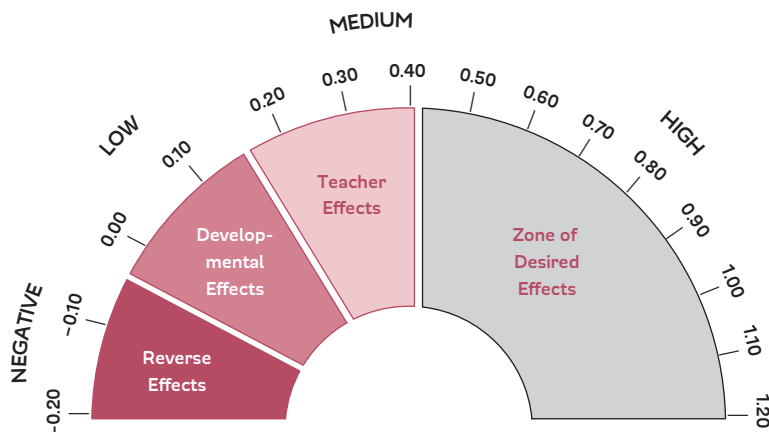
As we begin planning for our first-period algebra class or our afternoon geometry class, knowing the effect size of different influences, strategies, actions, and approaches to teaching and learning proves helpful in deciding where to devote our planning time and resources. Is a particular approach (e.g., classroom discussion, exit tickets, the use of calculators, jigsaw, computer-assisted instruction, creating simulations, cooperative learning, instructional technology, presenting clear success criteria, developing a rubric, etc.) worth the effort for the desired learning outcomes of that day, week, or unit? John Hattie was able to demonstrate that influences, strategies, actions, and approaches with an effect size greater than 0.40 allow students to learn at an appropriate rate, meaning *at least* a year of growth for a year in school. Effect sizes greater than 0.40 mean more than a year of growth for a year in school. Figure I.1 provides a visual representation of the range of effect sizes calculated in the Visible Learning research.

Before this level was established, teachers and researchers did not have a way to determine an acceptable threshold; thus, we continued to use weak practices, often supported by studies with statistically significant findings.

Consider the following examples. First, let us consider classroom discussion. Should teachers devote resources and time to planning for the facilitation of classroom discussion? Will this approach to mathematics provide a return on investment rather than “chalk talk,” where we work out lots of problems on the board for them to include in their notes? With classroom discussion, teachers intentionally design and purposefully plan for learners to talk with their peers about specific problems or approaches to problems (e.g., comparing approaches to solving a quadratic, completing the square or using the quadratic formula) in collaborative groups. Peer groups might engage in working to solve complex problems or tasks (e.g., data analysis, geometric proofs, maximization problems, or solving systems of equations in an authentic context). The students would not

EFFECT SIZE  
FOR CLASSROOM  
DISCUSSION = 0.82

# THE BAROMETER OF INFLUENCE



Source: Adapted from Hattie, J. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. Figure 2.4, page 19. New York, NY: Routledge.

Figure I.1

be **ability grouped** (tracking or streaming), but rather grouped by the teacher to ensure academic diversity in each group as well as language support and varying degrees of interest and motivation. As can be seen in the barometer in Figure I.2, the effect size of classroom discussion is 0.82, well above our threshold of accelerated learning gains.

Therefore, someone teaching mathematics in the Visible Learning classroom would use classroom discussion to understand mathematics learning through the eyes of their students and for students to see themselves as their own mathematics teachers. Talking about mathematics content and practices or processes helps us see learning through the eyes of our students and allows them to see themselves as their own teachers.

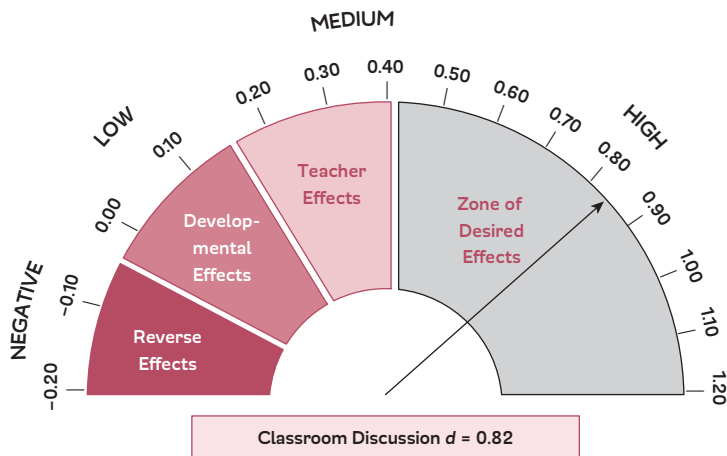
Second, let us look at the use of calculators. Within academic circles, teacher workrooms, school hallways, and classrooms, there have been many conversations about the use of the calculator in mathematics. There have been many efforts to reduce the reliance on calculators and the development of technology-enhanced items on assessments in mathematics. Using a barometer as a visual representation of effect sizes, we see that the use of calculators has an effect size of 0.27. The barometer for the use of calculators is in Figure I.3.

EFFECT SIZE FOR ABILITY GROUPING = 0.12

EFFECT SIZE FOR RESPONSE TO INTERVENTION = 1.29

**Ability grouping** is the long-term grouping or tracking of learners based on their ability. This is different from grouping students to work on a specific concept, skill, or application or to address a misconception.

# THE BAROMETER FOR THE INFLUENCE OF CLASSROOM DISCUSSION



Source: Adapted from Hattie, J. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. Figure 2.4, page 19. New York, NY: Routledge.

Figure 1.2

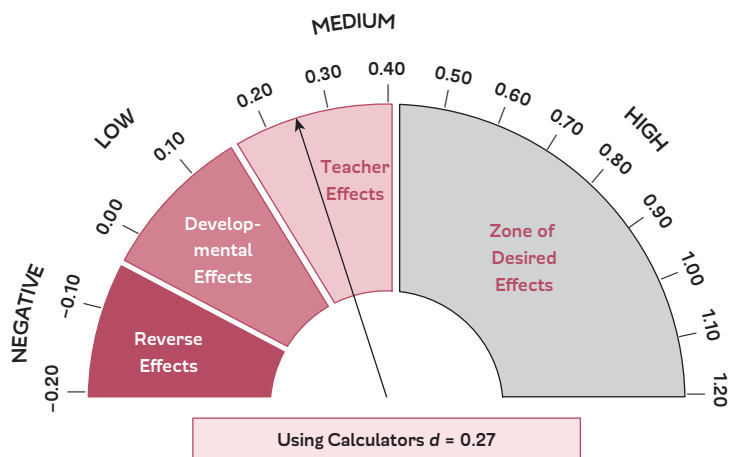
EFFECT SIZE  
OF USE OF  
CALCULATORS  
= 0.27

As you can see, the effect size of 0.27 is below the zone of desired effects of 0.40. The evidence suggests that the impact of the use of calculators on mathematics achievement is low. However, closer examination of the five meta-analyses and the 222 studies that produced an overall effect size of 0.27 reveals a deeper story to the use of calculators. Calculators are most effective in the following circumstances: (a) when they are used for computation, deliberate practice, and learners checking their work; (b) when they are used to reduce the amount of cognitive load on learners as they engage in problem solving; and (c) when there is an intention behind using them (e.g., solving by graphing or approximation problems). This leads us into a second key finding from John Hattie's *Visible Learning* research: *We should not hold any influence, instructional strategy, action, or approach to teaching and learning in higher esteem than students' learning.*

## What Works Best When

Visible Learning in the mathematics classroom is a continual evaluation of our impact on student learning. Regarding the calculator example, their use is not really the issue and should not be our focus. Instead, our focus should be on the intended learning outcomes for that day and how calculators support that learning. Visible Learning is

# THE BAROMETER FOR THE INFLUENCE OF USING CALCULATORS



Source: Adapted from Hattie, J. (2009). *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. Figure 2.4, page 19. New York, NY: Routledge.

Figure I.3

more than a checklist of dos and don'ts. Rather than checking influences with high effect sizes off the list and scratching out influences with low effect sizes, we should match the *best* strategy, action, or approach with the learning needs of our students. In other words, is the use of calculators the right strategy or approach for the learners at the right time for this specific content? Clarity about the learning intention brings into focus what the learning should be for the day, why students are learning about this particular piece of content and process, and how we *and* our learners will know they have learned the content. Teaching mathematics in the Visible Learning classroom is not about a specific strategy, but a location in the learning process.

Over the next several chapters, we will show how to support mathematics learners in their pursuit of conceptual understanding, procedural knowledge, and application of concepts and thinking skills through the lens of *what works best when*. This requires us, as mathematics teachers, to be clear in our planning and preparation for each learning experience and challenging mathematics tasks. Using guiding questions, we will model how to blend what works best with what works best *when*. You can use Figure I.4 in your own planning. This is also found in Appendix B and online at [resources.corwin.com/vlmathematics-9-12](https://resources.corwin.com/vlmathematics-9-12).

## Teaching Takeaway

Using the right approach at the right time increases our impact on student learning in the mathematics classroom.

# HOW TO USE APPENDIX B WHEN PLANNING FOR CLARITY

I have to be clear about what content and practice or process standards I am using to plan for clarity. Am I using only mathematics standards or am I integrating other content standards (e.g., writing, reading, or science)?

Rather than what I want my students to be doing, this question focuses on the learning. What do the standards say my students should learn? The answer to this question generates the **learning intentions** for this particular content.

Once I have clear learning intentions, I must decide when and how to communicate them with my learners. Where does it best fit in the instructional block to introduce the day's learning intentions? Am I going to use guiding questions?

As I gather evidence about my students' learning progress, I need to establish what they should know, understand, and be able to do that would demonstrate to me that they have learned the content. This list of evidence generates the **success criteria** for the learning.

## ESTABLISHING PURPOSE

1

**What are the key content standards I will focus on in this lesson?**

Content Standards:

2

**What are the learning intentions (the goal and why of learning, stated in student-friendly language) I will focus on in this lesson?**

Content:

Language:

Social:

3

**When will I introduce and reinforce the learning intention(s) so that students understand it, see the relevance, connect it to previous learning, and can clearly communicate it themselves?**

## SUCCESS CRITERIA

4

**What evidence shows that students have mastered the learning intention(s)? What criteria will I use?**

I can statements:



This planning guide is available for download at [resources.corwin.com/vlmathematics-9-12](https://resources.corwin.com/vlmathematics-9-12).



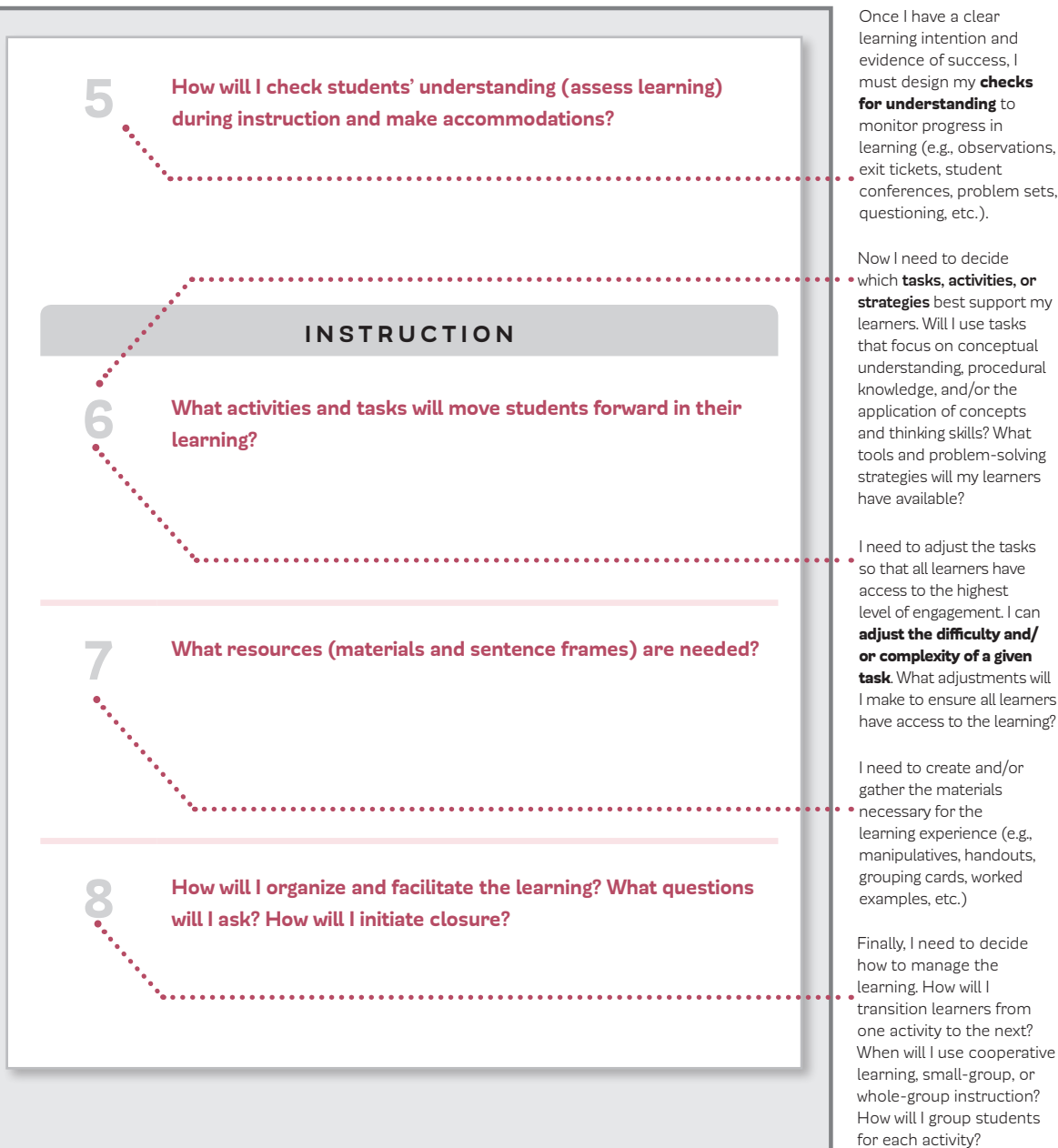


Figure I.4

Through these specific, intentional, and purposeful decisions in our mathematics instruction, we pave the way for helping learners see themselves as their own teachers, thus making them assessment-capable visible learners in mathematics.

## The Path to Assessment-Capable Visible Learners in Mathematics

Teaching mathematics in the Visible Learning classroom builds and supports assessment-capable visible learners (Frey, Hattie, & Fisher, 2018). With an effect size of 1.44, providing a mathematics learning environment that allows learners to see themselves as their own teacher is essential in today's classrooms. The QR code in the margin provides a glimpse of two collaborative mathematics classrooms. In both classrooms, the teachers work together to deliberately, intentionally, and purposefully provide learning experiences that build and support assessment-capable visible learners. Through effective co-teaching, these teachers provide *all* learners access to rich mathematics learning.

The following characteristics apply to assessment-capable visible mathematics learners:

1. They are active in their mathematics learning. Learners deliberately and intentionally engage in learning mathematics content and practices or processes by asking themselves questions, monitoring their own learning, and taking the reins of their learning. They know their current level of learning.

An assessment-capable visible learner says, "I am comfortable finding the simultaneous solution for a system of equations using graphing but need more learning on the elimination and substitution approach. I know there are examples in my interactive notebook that I can use to prepare for tomorrow's challenge problem."

2. They are able to plan the next steps in their progression toward mastery in learning mathematics content. Because of the active role taken by an assessment-capable visible mathematics learner, these students can plan their next steps and select the right tools (e.g., manipulatives, problem-solving approaches, and/or meta-cognitive strategies) to support working toward given learning

EFFECT SIZE FOR  
ASSESSMENT-  
CAPABLE VISIBLE  
LEARNERS = 1.33

intentions and success criteria in mathematics. For example, a student might respond to feedback, saying, “There is a more efficient way to solve this quadratic equation. I am going to use completing the square this time to see if I can find a more precise answer.” They know what additional tools they need to successfully move forward in a task or topic.

An assessment-capable visible learner says, “To find the solution to the system of equations, I am going to use substitution. Looking at the graph of this system of equations, the solution does not appear to be a pair of integers. Substitution will allow me to find a more accurate and precise solution.”

3. They are aware of the purpose of the assessment and feedback provided by peers and the teacher. Whether the assessment is informal, formal, formative, or summative, assessment-capable visible mathematics learners have a firm understanding of the information behind each assessment and the feedback exchanged in the classroom. Put differently, these learners not only seek feedback, but they recognize that errors are opportunities for learning, monitor their progress, and adjust their learning (adapted from Frey et al., 2018) (see Figure I.5).

An assessment-capable visible learner says, “Yesterday’s exit ticket surprised me. Ms. Norris wrote on my paper that I needed to revisit the process for isolating  $x$  and then substituting the expression into the second equation. So today I am going to work out the entire process in my notebook and try not to skip steps or do parts of the process in my head.”

Over the next several chapters, we will explore how to create a classroom environment that focuses on learning and provides the best environment for developing assessment-capable visible mathematics learners who can engage in the mathematical habits of mind represented in one form or another in every standards document. Such learners can achieve the following:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.



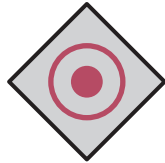
## Video 2

### Creating Assessment-Capable Visible Learners

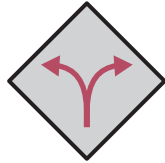
<https://resources.corwin.com/v/mathematics-9-12>

# ASSESSMENT-CAPABLE VISIBLE LEARNERS

## ASSESSMENT-CAPABLE LEARNERS:



**KNOW** THEIR CURRENT LEVEL OF UNDERSTANDING



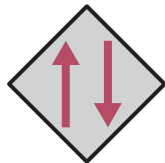
**KNOW** WHERE THEY'RE GOING AND ARE CONFIDENT TO TAKE ON THE CHALLENGE



**SELECT** TOOLS TO GUIDE THEIR LEARNING



**SEEK** FEEDBACK AND RECOGNIZE THAT ERRORS ARE OPPORTUNITIES TO LEARN



**MONITOR** THEIR PROGRESS AND ADJUST THEIR LEARNING



**RECOGNIZE** THEIR LEARNING AND TEACH OTHERS

Source: Adapted from Frey, Hattie, & Fisher (2018).

Figure I.5

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (© Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.).

## How This Book Works

As authors, we assume you have read *Visible Learning for Mathematics* (Hattie et al., 2017), so we are not going to recount all of the information contained in that book. Rather, we are going to dive deeper into aspects of high school mathematics instruction that are critical for students' success, helping you to envision what a Visible Learning mathematics classroom like yours looks like. In each chapter, we profile three high school teachers who have worked to make mathematics learning visible for their students and have influenced learning in significant ways. Each chapter will do the following:

1. Provide effect sizes for specific influences, strategies, actions, and approaches to teaching and learning.
2. Provide support, through research, for specific strategies and approaches to teaching mathematics.
3. Incorporate content-specific examples from secondary mathematics curricula.
4. Highlight aspects of assessment-capable visible learners.

Through the eyes of algebra, geometry, and statistics teachers, as well as the additional secondary mathematics teachers in the accompanying videos, we aim to show you the mix and match of strategies you can use to orchestrate your lessons in order to help your students build their conceptual understanding, procedural fluency, and application of concepts and thinking skills in the most visible ways possible—visible to you and to them. As you may have noticed, you will see instances of classrooms that use a collaborative teaching situation. While some of the co-teachers have a special education background, it is important to

note that the teachers work as equal collaborative partners who are there to support all learners. They plan together, they teach together, and they evaluate their impact on student learning together. Teaching mathematics in a Visible Learning classroom *can be done* with all students and in any classroom. If you're a mathematics specialist, mathematics coordinator, or methods instructor, you may be interested in exploring the vertical progression of these content areas preK–12 within Visible Learning classrooms and see how visible learners grow and progress across time and content areas. While you may identify with one of the teachers from a content perspective, we encourage you to read all the vignettes to get a full sense of the variety of choices you can make in your instruction, based on your instructional goals.

In the first chapter, we focus on the aspects of mathematics instruction that must be included in each lesson. We explore the components of effective mathematics instruction (conceptual, procedural, and application) and note that there is a need to recognize that student learning has to occur at the surface, deep, and transfer levels within each of these components. Surface, deep, and transfer learning served as the organizing features of *Visible Learning for Mathematics*, and we will briefly review them and their value in learning. Finally, Chapter 1 contains information about the use of checks for understanding to monitor student learning. Generating evidence of learning is important for both teachers and students in determining the impact of the learning experiences and challenging mathematical tasks on learning. If learning is not happening, then we must make adjustments.

Following this introductory chapter, we turn our attention, separately, to each component of mathematics teaching and learning. However, we will walk through the process, starting with the application of concepts and thinking skills, then direct our attention to conceptual understanding, and finally, procedural knowledge. This seemingly unconventional approach will allow us to start by making the goal or endgame visible: learners applying mathematics concepts and thinking skills to other situations or contexts.

Chapter 2 focuses on *application* of concepts and thinking skills. Returning to our three profiled classrooms, we will look at how we plan, develop, and implement challenging mathematical tasks that scaffold student thinking as students apply their learning to new contexts or

situations. Teaching mathematics in the Visible Learning classroom means supporting learners as they use mathematics in a variety of situations. Returning to Figure I.4, we will walk through the process for establishing clear learning intentions, defining evidence of learning, and developing challenging tasks that, as you already have come to expect, encourage learners to see themselves as their own teachers. The final section of this chapter will focus on how to differentiate mathematical tasks by adjusting their difficulty and/or complexity, working to meet the needs of all learners in the mathematics classroom.

Chapter 3 and Chapter 4 take a similar approach with conceptual understanding and procedural knowledge, respectively. Using Chapter 2 as a reference point, we will return to the three profiled classrooms and explore the conceptual understanding and procedural knowledge that provided the foundation for their learners applying ideas to different mathematical situations. For example, what influences, strategies, actions, and approaches support a learner's conceptual understanding of systems of equations, the unit circle, or inferential statistics? As in Chapter 2, we will talk about differentiating tasks by adjusting the difficulty and complexity of these tasks.

In this book, we do not want to discourage the value of procedural knowledge. Although mathematics is more than procedural knowledge, developing skills in basic procedures is needed for later work in each area of mathematics, from complex numbers to conditional probability. As in the previous two chapters, Chapter 4 will look at what works best when in supporting students' fluency in procedural knowledge. Adjusting the difficulty and complexity of tasks will once again help us meet the needs of all learners.

In the final chapter of this book, we focus on how to make mathematics learning visible through evaluation. Teachers must have clear knowledge of their impact so that they can adjust the learning environment. Learners must have clear knowledge about their own learning so that they can be active in the learning process, plan the next steps, and understand what is behind the assessment. What does evaluation look like so that teachers can use it to plan instruction and to determine the impact they have on learning? As part of this chapter, we highlight the value of feedback and explore the ways in which teachers can provide effective feedback to students that is growth producing. Furthermore,

we will highlight how learners can engage in self-regulation feedback and provide feedback to their peers.

This book contains information on critical aspects of secondary mathematics instruction that have evidence for their ability to influence student learning. In the appendices, we provide additional resources for implementing these critical aspects of secondary mathematics instruction. We're not suggesting that these be implemented in isolation, but rather that they be combined into a series of linked learning experiences that result in students engaging in mathematics learning more fully and deliberately than they did before. Whether calculating slope or the area under the curve, we strive to create a mathematics classroom where we *see* learning through the eyes of our students and students *see* themselves as their own mathematics teachers. As learners progress from simplifying rational expressions to solving related rates, teaching mathematics in the Visible Learning classroom should develop and support assessment-capable visible mathematics learners.