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Introduction to Curriculum Topic Study

Curriculum Topic Study was extremely helpful as a cornerstone and starting point for our lesson study. As we planned our lesson, we used Curriculum Topic Study to stay grounded in the content goals for the lesson and identify potential learning difficulties.

Curriculum Topic Study has opened my eyes as to the meaning of adopting a "standards-based" program. I frequently refer to the Curriculum Topic Study Guides to help me find the information I need to understand concepts that I previously would not have considered teaching at my grade level.

Our math department meetings have changed significantly since using Curriculum Topic Study to build a common understanding and language about our state standards. Using the resources and discussing our findings has given us the confidence to make sound decisions about our common assessments and instructional strategies.

Each time our K–8 curriculum committee studies a new topic, I leave with a greater understanding of how student ideas build over time and the importance of a coherent curriculum that connects mathematical ideas.

Curriculum Topic Study has helped me design workshops for curriculum implementation support. Using Curriculum Topic Study as a lens to examine our new Connected Mathematics units has helped our teachers understand how the lessons align with standards and flow conceptually. In addition, Curriculum Topic Study has given our teachers added confidence and tools to improve their own understanding of mathematical ideas.

WHAT IS CURRICULUM TOPIC STUDY?

The above quotes came from mathematics educators who use Curriculum Topic Study (CTS) in their work as teachers, mentors, professional developers, and curriculum and assessment committee members and as part of collaborative learning communities. CTS is a methodical study process that uses a set of tools and strategies—organized around 92 curriculum topics—and is designed to help educators improve the teaching and learning of mathematics. Mathematics educators who use CTS in a deliberate and systematic way will:

- Develop a deeper understanding of the specific content they teach
- Identify and clarify the specific concepts and procedures that are important for students to know and be able to do
- Clarify learning goals from their state or local standards
- Gain a deeper understanding of the intent of national, state, and local mathematics standards
- Improve the coherency of development within and across grades and topics
- Identify potential learning difficulties, developmental considerations, misconceptions, and misunderstandings associated with a topic
- Examine essential characteristics of effective instruction and models of standards- and research-based practice
- Apply effective strategies for teaching specific ideas and procedures associated with a topic
- Take advantage of important connections between mathematics and science instruction
- Increase opportunities for students of all levels and backgrounds to achieve the learning goals articulated in district, state, and national standards
- Acquire and use a common language and knowledge base about teaching and learning in mathematics
- Provide more content focus to professional development activities

CTS helps individuals or groups of teachers study, in a systematic way, relevant readings from a core set of professional mathematics education resources. These readings are identified and screened in advance, then organized in Curriculum Topic Study Guides (see example in Figure 1.1). The specific features and uses of CTS Guides are described in Chapter 2.

There are 92 CTS Guides in Chapter 6—ranging from specific topics, such as triangles, to broader topics, such as geometric shapes—organized in seven categories. The majority of the guides address K–12 topics. Guides that address sophisticated content, such as quadratics, are designed to be used primarily with upper grade levels.

For each topic, a CTS Guide lists relevant readings from sources that include national standards documents, trade books written by mathematicians, research summaries, and K–12 conceptual strand maps (see Figure 1.2). Optional readings, videos, and Web-based material can also be used to supplement the study of a topic.

Figure 1.1 Example of a Curriculum Topic Study Guide

Standards and Research-Based Study of a Curricular Topic

PROBABILITY

Section and Outcome	**Selected Sources and Readings for Study and Reflection** Read and examine *related parts* of:
I. Identify Adult Content Knowledge	**IA:** ***Science for All Americans*** ▸ Chapter 9, *Probability,* pages 135–137 **IB:** ***Beyond Numeracy*** ▸ *Coincidences,* pages 38–41 ▸ *Probabilities,* pages 187–191 ▸ *Statistics – Two Theorems,* pages 227–230
II. Consider Instructional Implications	**IIA:** ***Benchmarks for Science Literacy*** ▸ 9D, *Uncertainty,* general essay page 226, grade span essays, pages 227–230 **IIB:** ***NCTM Principles and Standards for School Mathematics*** ▸ Grades PreK–12 Overview Data Analysis and Probability, page 48, *Understand and Apply*, page 51 ▸ Grades PreK–2 Data Analysis and Probability, page 109, *Understand and Apply*, page 114 ▸ Grades 3–5 Data Analysis and Probability, *Understand and Apply*, page 181 ▸ Grades 6–8 Data Analysis and Probability, *Understand and Apply*, pages 253–255 ▸ Grades 9–12 Data Analysis and Probability, page 325, *Understand and Apply*, pages 331–333
III. Identify Concepts and Specific Ideas	**IIIA:** ***Benchmarks for Science Literacy*** ▸ 9D, *Uncertainty,* pages 227–230 **IIIB:** ***NCTM Principles and Standards for School Mathematics*** ▸ Grades PreK–2 Data Analysis and Probability, page 108 or 400 ▸ Grades 3–5 Data Analysis and Probability, page 176 or 400 ▸ Grades 6–8 Data Analysis and Probability, page 248 or 401 ▸ Grades 9–12 Data Analysis and Probability, page 324 or 401
IV. Examine Research on Student Learning	**IVA:** ***Benchmarks for Science Literacy*** ▸ 9D, *Probability,* page 353 **IVB:** ***Research Companion*** ▸ Chapter 14, *Research on Students' Understanding of Probability*, pages 216–224
V. Examine Coherency and Articulation	**V:** ***Atlas of Science Literacy*** ▸ *Statistical Reasoning,* page 127 noting the conceptual strand "Probability"
VI. Clarify State Standards and District Curriculum	**VIA:** ***State Standards:*** Link Sections I–V to learning goals and information from your state standards or frameworks that are informed by the results of the topic study. **VIB:** ***District Curriculum Guide:*** Link Sections I–V to learning goals and information from your district curriculum guide that are informed by the results of the topic study.

Visit www.curriculumtopicstudy.org for updates or supplementary readings, Web sites, and videos.

Figure 1.2 Types of Readings and Their Sources

Type of Resource	Source
Adult mathematics literacy description	*Science for All Americans,* AAAS Project 2061 (AAAS, 1990)
Adult mathematics trade book	*Beyond Numeracy,* by John Allen Paulos (1992)
National, state, and local standards	*Benchmarks for Science Literacy,* AAAS Project 2061, (AAAS, 1993) *Principles and Standards for School Mathematics* (NCTM, 2000) State standards or frameworks and/or local curriculum standards or frameworks
Research summaries	*Benchmarks for Science Literacy,* Chapter 15, Project 2061 *Research Companion to Principles and Standards for School Mathematics* (NCTM, 2003)
Conceptual strand maps	*Atlas of Science Literacy,* American Association for the Advancement of Science Project 2061, AAAS (2001)

CTS users may find additional material to complement the readings in a selected CTS Guide from their own collection of professional resources or on the CTS Web site at www.curriculumtopicstudy.org.

CTS provides an effective and efficient way to intellectually engage with mathematics professional readings. Typically, educators have to sift through a huge amount of unfamiliar and often daunting material from disparate sources. Indeed, many teachers may not even know where to look for the information they are seeking. CTS Guides identify the purpose of different resources and explicitly link relevant parts of the text information contained in the resources to topics of study that are useful from the teachers' perspective.

> CTS Guides do the groundwork for the busy educator, providing a one-page guide to relevant results from an enormous range of readings that have been vetted and organized in advance.

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000), *Science for All Americans* (American Association for the Advancement of Science [AAAS], 1990), and *Benchmarks for Science Literacy* (AAAS, 1993) have provided a carefully crafted description of the mathematical ideas and skills all students should achieve by the time they graduate from high school. *Atlas of Science Literacy* (AAAS, 2001) further clarified those documents by providing a set of conceptual strand maps that detail how those ideas and skills connect and develop from kindergarten through high school. States and districts have modeled their own standards after these national documents, and stakeholders at every level are learning how to evaluate, modify, and develop assessments, curricula, and instructional materials to reflect this vision of mathematics for all students.

> What has been missing is a *comprehensive, systematic* process to help educators make effective use of national standards and research on student learning.

Although few mathematics teachers realize that *Science for All Americans, Benchmarks,* and the *Atlas* address student learning in mathematics as well as science, many teachers have seen or own a copy of the *Principles and Standards.* Also, the research on student learning is growing and becoming increasingly accessible to practitioners through print publications and on the Web. Yet, for all of the thought that went into national standards and all of the research on how students learn, just having the documents and research articles is not enough to truly impact student learning. In particular, this means being able to relate them to state and local standards and to their own curricular and instructional challenges. CTS is a *deliberate* process that uses a set of common, high-quality, collective resources to help mathematics educators become better informed educators who understand what the label "standards- and research-based" means.

CTS is not a replacement for formal content coursework, but it can help teachers learn new content or refresh their content knowledge at the same time that they are studying the pedagogical implications of teaching that content. This can be particularly helpful to elementary teachers, who are expected to teach all content areas and seldom have substantive coursework in mathematics. But it can also be helpful to teachers who have had upper-level mathematics coursework, helping them translate formal mathematics content into content that is appropriate for students at different grade levels. This synthesis of content and pedagogy is an essential learning experience for teachers, an experience that content courses and many professional development offerings largely overlook.

> CTS helps educators make the bridge between national standards and research and their local and state efforts to help all students learn challenging mathematics.

Educators who use CTS will realize what powerful tools the national standards and research on student learning can be for implementing their state standards and improving their students' opportunities to learn mathematics. And they will know how to use those tools effectively. CTS provides guidance and effective strategies for educators to use professional resources and research in their practice. CTS moves the mathematics standards off the shelf and into the hands and minds of teachers, leaders, and professional developers, who can use them routinely in an effective, systematic way to improve teaching and learning of mathematics.

WHY STUDY A CURRICULUM TOPIC?

By taking the time to study a topic before planning a unit or lesson, teachers build a deeper understanding of the content, connections, and effective ways to help students achieve understanding of the most important concepts and procedures in that topic. This can significantly improve instruction, regardless of the materials used. A former director of the National Science Foundation's (NSF) Elementary, Secondary, and Informal Education Division points out that teachers who are unfamiliar with the topics they teach tend to rely on textbooks and teach in a more didactic way, often failing to make connections between important ideas in mathematics:

> . . . [When] teachers cover topics about which they are well-prepared, they encourage student questions and discussions, spend less time on unrelated topics, permit discussions to move in new directions based on student interest, and generally present topics in a more coherent way, all strategies described as standards-based teaching. However, when teachers teach topics about which they are less well-informed, they often discourage active participation by students, keep any discussion under tight rein, rely more on presentation than on student discussions, and spend time on tangential issues. (Kahle, 1999)

National standards, research, and other professional tools can help teachers prepare in ways that allow them to provide the best learning experiences possible for their students. But teachers need to see the connection between those resources and tools and the topics they teach. CTS makes that connection, ensuring that those resources and tools are used in ways that are effective, meaningful, and relevant to the teacher.

A plethora of general professional tools for teachers are available in schools, districts, and professional development settings. Examples include: *Concept-Based Curriculum and Instruction* (Erickson, 1998), *Understanding by Design* (Wiggins & McTighe, 1998), *Enhancing Professional Practice: A Framework for Teaching* (Danielson, 1996), *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement* (Marzano, Pickering, & Pollack, 2001), *Mapping the Big Picture: Integrating Curriculum and Assessment K–12* (Jacobs, 1997), and *Differentiated Instructional Strategies: One Size Doesn't Fit All* (Gregory & Chapman, 2002). Such tools can be useful to teachers who know the content and structure of their discipline and are familiar with the research base on student learning. But the same tools may not suffice for novice teachers, elementary teachers who teach all content areas, and others who may not have a sufficient knowledge base in the area of mathematics they teach. Current reform-oriented practices in mathematics center on teaching for understanding and engaging students in the "big ideas" of mathematics. As a result, "one tool fits all" may result in "one tool fits few" if we fail to help the teacher connect general teaching resources to the specific ideas and skills they are actually teaching.

CTS extends the power of these valuable although content-generic tools. While CTS is right in line with their underlying principles, it also provides a topic-specific study process needed to make these tools more effective for use by mathematics teachers. For example, CTS builds on current thinking about designing learning experiences in approaches such as *backwards design*, used in *Understanding by Design* (Wiggins & McTighe, 1998). Backwards design begins by identifying evidence of meeting desired standards and then proceeds to plan teaching and learning experiences. This model suggests four filters for determining what is worth teaching and understanding in a topic. Three of these filters relate to the CTS process and include examining (1) the extent to which the "big ideas" in a topic are addressed, (2) the extent to which the ideas and processes in a topic reside at the heart of the discipline, and (3) the extent to which abstract and counterintuitive ideas need to be uncovered, including students' misconceptions about ideas related to the topic.

Although these three filters can be powerful, they assume that teachers are comfortable with mathematical content, know what the "big ideas" are, can make the connections that support student learning, and are aware of students' misconceptions and alternative ways of thinking. Unfortunately, this often is not the case. In reality, many good teachers missed the "big ideas" in their own mathematics education. Elementary and middle school teachers may have a limited mathematics background, and high school mathematics teachers frequently specialize in a particular area of mathematics. Teachers may not be aware of the misconceptions and alternative ideas their students hold, and sometimes, they harbor those very same misconceptions.

> CTS combines the wisdom of teacher practice with the recommendations from standards and research and serves as the essential first step in effective planning and design.

The critical and often overlooked first step in any effective instructional design process involves a clear understanding of the specific ideas and procedures that students need to learn and the pedagogical implications of helping students learn them. A careful study of a topic, using CTS, clarifies the "end in mind" and provides a framework for planning assessment and instruction that is true to the content and takes student thinking and developmental levels into account. All too often, teachers strike out on their own, even when a wealth of information and resources, carefully thought through by distinguished mathematicians and educators, sits at their fingertips.

CTS also reflects the research base on teacher knowledge and the importance of pedagogical content knowledge. Knowing the content is different from knowing how to organize and represent it in a way that encourages student learning. Indeed, an advanced understanding of content can sometimes make it harder to identify difficulties a novice learner is likely to have. Designing learning experiences and facilitating learning requires pedagogical knowledge related to mathematics content. Teachers with this special knowledge understand what makes the learning of specific topics easy or difficult for learners and can develop strategies for representing and formulating content to make it accessible to learners (Shulman, 1986). Thus, even those teachers who understand the mathematics behind the topics they teach can benefit from CTS. Through CTS, teachers with a strong mathematics content background can gain new insights about specific ideas and procedures that may have been overlooked, connections within and across topics, effective contexts for learning, developmental considerations for introducing new ideas and skills, and new instructional strategies that result in increased learning.

Outside of the work done by NCTM through regional and annual meetings, academies, and the distribution of outreach material; and AAAS's support for K–12 science literacy (which includes mathematics), there has been little systematic, widespread work to help teachers understand and use standards and research on student learning. Many pre- and inservice efforts to support teachers' content knowledge place little emphasis on helping teachers become aware of the connection between the topics they teach, the recommendations in standards, and research on students' alternative ideas and ways of thinking. Without a process to compare current practice with standards and research, teachers are likely to continue doing what they have always been doing. Recall the old adage, "If you always do what you've always done, you'll always get what you've always gotten."

> Teachers may have copies of standards or occasionally come across a research article; what has been missing is a process for using them to impact teaching practice, knowledge, and beliefs.

CTS brings this process to teachers at all levels of experience and engages them in classroom applications of standards and research. In today's climate of accountability, teachers are assuming more personal responsibility for their own learning. Rather than waiting for the system to provide more effective professional development programs, mathematics educators can use CTS to continue to grow and improve as teachers, enhancing student learning of the most important ideas in mathematics.

WHY FOCUS ON TOPICS?

To understand why this book focuses on topics, it is important to clarify what is meant by the word. In the context of CTS, *topics* are the broad organizers for ideas and skills in a curriculum; they do not describe the endpoint of instruction. Learning goals describe what students should know and be able to do after instruction, but they need to be organized and sorted into important topics within the curriculum (see Figure 1.3). Unfortunately, the research and the national, state, and local

Figure 1.3 Topics as Organizers

Topics as Organizers for Curriculum and Instruction

- Teachers design instruction by organizing the mathematics concepts and procedures they teach by unit topics.
- Curriculum materials and district curriculum guides are often organized by topics. Yet, the standards and results of research describing the cognitive difficulties students face are organized by broad conceptual strands.
- Topics provide a framework for understanding the nature of mathematics, connections among mathematical ideas, problem-solving processes, applications to other disciplines, and implications of mathematics and technology.

standards that drive curriculum, instruction, and assessment often are not organized in the same way as the mathematics content in school curricula. Educators must make a bridge between the way they organize mathematics topics and the specific, research-based ideas and procedures laid out in national, state, and local standards. CTS provides a methodical way to make this bridge.

> Because curriculum and instruction are organized around topics, CTS uses topics as the entry point for looking at important and related ideas and procedures taught within a single lesson or a broader curricular unit.

Making this distinction clear, while highlighting the relationship between learning goals and topics, is one of the main goals of CTS. Rather than focusing on teaching a topic per se, CTS helps teachers think about the organization of curriculum, instruction, and assessment around a connected set of specific ideas and skills. When teachers focus solely on topics without drilling down into the specific key ideas, they risk losing the benefits of standards and research, which make critical distinctions within topics about what is important to teach and how best to teach it. Topics provide a framework for understanding the nature of mathematics, connections among mathematical ideas, problem-solving processes, applications to other disciplines, and implications of mathematics and technology.

CTS Guides range in grain size from a specific idea or skill, such as addition and subtraction, to a broader topic, such as computation and operations (within which addition and subtraction is a subtopic). For a topic to be included in a CTS Guide, it must be considered critical content in mathematics, and it must be linked to specific learning goals articulated in national standards.

There are 92 topics included in this book (see Chapter 6), but this does not imply that all of these topics should be taught in the K–12 mathematics curriculum. The Third International Mathematics and Science Study suggests that American curricula suffer from being a "mile wide and an inch deep," resulting in little conceptual understanding for students (Schmidt, McKnight, & Raizen, 1997). CTS is meant to help teachers provide students with a deeper conceptual understanding of the learning goals in a selected topic, not to ensure that they cover every possible topic superficially.

The intent of CTS is to provide enough examples of common topics so that teachers can find CTS Guides that address the topics they are currently teaching, consolidate topics for deeper understanding, and improve their understanding of how students best learn the ideas and skills in the topics they teach. To this end, the CTS Guides include both traditional topics that might appear in textbooks, standards-based curriculum materials, district scope and sequence curriculum guides, and content categories used to organize national and state standards and frameworks. Many of these topics have overlapping concepts and procedures, and some are completely

Figure 1.4 Characteristics of Expert Teachers

Expert Teachers

- Know the structure of the knowledge in their disciplines
- Know the conceptual barriers that are likely to hinder learning
- Have a well-organized knowledge of concepts and inquiry procedures and problem-solving strategies (based on pedagogical content knowledge)

SOURCE: Bransford et al., 2000.

subsumed in others. Furthermore, because each CTS Guide examines a topic from a K–12 perspective, teachers eliminate unnecessary redundancy while planning purposeful reiteration of ideas that need to be revisited in different contexts or at increasing levels of sophistication. This careful examination of a topic, in contrast to the "checklist of standards or objectives," promotes the coherence that may be lacking in current attempts at standards-based curriculum, instruction, and assessment.

THE UNDERLYING KNOWLEDGE AND RESEARCH BASE

Teachers need to develop a personal understanding of the reform recommendations articulated in *Principles and Standards for School Mathematics* and other documents. Clearly, translating standards into classroom practice is a challenge yet to be overcome. At the same time, there is a shift toward providing transformative professional development and supporting resources that reflect the current knowledge base on how teachers and students learn. The NRC reports, *How People Learn* (Bransford, Brown, & Cocking, 2000) and *How Students Learn Mathematics in the Classroom* (Donovan & Bransford, 2005) are raising awareness among mathematics educators of the need to understand the preconceptions and alternative ideas students bring to their learning. CTS—by virtue of its focus on the structure of mathematics content, research into students' learning, and pedagogical strategies linked to specific ideas and skills—reflects the findings from *How People Learn,* which distinguish expert teachers from novices (see Figure 1.4). There is a strong link between teacher expertise, which involves both content and pedagogical content knowledge, and student achievement. Because teacher expertise has such a demonstrated impact on student learning, it stands to reason that processes that develop mathematics teachers' knowledge and skills, such as CTS, are a sound investment toward improving student achievement in mathematics.

National standards have been around for almost a decade, yet studies such as *Investigating the Influence of Standards* (National Research Council [NRC], 2002) show that standards have not made a significant impact where it matters most—the classroom.

THE ORIGIN OF CURRICULUM TOPIC STUDY: FROM SCIENCE TO MATHEMATICS

The CTS approach was first adapted for use in science education from AAAS Project 2061's study of a benchmark, a powerful approach to understanding the intent of a specific goal for student learning and seeing how that learning goal can impact

Figure 1.5 Parallel Resources Used in Science and Mathematics Curriculum Topic Studies (CTS)

Science CTS Resources	**Mathematics CTS Resources**
Science for All Americans (AAAS, 1990)	*Science for All Americans* (AAAS, 1990)
Science Matters (Hazen & Trefil, 1991)	*Beyond Numeracy* (Paulos, 1992)
Benchmarks for Science Literacy (AAAS, 1993)	*Benchmarks for Science Literacy* (AAAS, 1993)
National Science Education Standards (National Research Council [NRC], 1996)	*Principles and Standards for School Mathematics* (NCTM, 2000)
Making Sense of Secondary Science (Driver, Squires, Rushworth, & Wood-Robinson, 1994)	*Research Companion to Principles and Standards for School Mathematics* (NCTM, 2003)
Atlas of Science Literacy (AAAS, 2001)	*Atlas of Science Literacy* (AAAS, 2001)
State standards, frameworks, or curriculum guides	State standards, frameworks, or curriculum guides

teachers' knowledge and practice. Although it is important to examine a single learning goal in certain cases, the authors' work with teachers throughout New England revealed that they often need to examine collections of learning goals and relate them explicitly to the topics they teach. It was also clear that teachers benefited from working with a range of standards documents, including both *Benchmarks for Science Literacy* (AAAS, 1993) and the *National Science Education Standards* (NRC, 1996), and from looking beyond standards documents to resources for learning content and research on how students learn specific ideas and skills. Based on these experiences, CTS expanded the Project 2061 benchmark study procedure to include examination of additional content resources such as *Science Matters: Achieving Scientific Literacy* (Hazen & Trefil, 1991), *Making Sense of Secondary Science* (Driver, Squires, Rushworth, & Wood-Robinson, 1994), and topic-specific supplementary resources such as videos, journal articles, Web sites, and content material.

Our experience suggested that mathematics teachers would benefit from a similar topic study approach. Draft versions of the mathematics CTS were developed to be used in the NSF-funded Northern New England Co-Mentoring Network (www.nnecn.org) to help mathematics mentors use national standards and cognitive research in their work with novice teachers. Subsequently, a mathematics version of *Science Curriculum Topic Study: Bridging the Gap Between Standards and Practice* (Keeley, 2005) underwent development, piloting, and national field-testing after receiving funding in 2004 from NSF's Teacher Professional Continuum Program. Figure 1.5 shows the parallel collection of resources used in science and mathematics CTS.

National, State, and Local Standards

The term *standards* (sometimes referred to as benchmarks, learning results, performance indicators, and so on) conveys different meanings to different people. CTS defines *standards* as a set of outcomes that individually define the mathematical ideas and procedures that students should know and be able to do and that collectively provide a vision for achieving mathematical literacy for all students. A misperception by many educators and school systems is that the standards themselves are, or imply, a curriculum. Curricular decisions at the state and local level should be *informed* by standards, but there is a variety of ways to organize the ideas and skills in standards into a curriculum.

The national content standards used in mathematics CTS include the *Principles and Standards for School Mathematics* and *Benchmarks for Science Literacy*. NCTM, an international professional organization committed to excellence in mathematics teaching and learning for all teachers and students, first developed standards for mathematics in 1989, including *Curriculum and Evaluation Standards for School Mathematics.* After 10 years of ongoing efforts by a group of teachers, teacher educators, administrators, researchers, and mathematicians to examine, monitor, and evaluate the NCTM standards, a revised and retitled document was released in 2000. The major components of *Principles and Standards for School Mathematics* establish a foundation for school mathematics programs and articulate content and process standards that describe what students should know and be able to do as they progress through the grades.

The NCTM documents were the first set of standards to promote mathematical literacy for all students. Mathematical literacy can be defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned, and reflective citizen" (De Lange, 1999, p. 76). Since the release of the first document, most states and local districts have developed their own content standards or are revising their existing content standards. In addition, multiple curriculum materials and instructional programs have been developed and aligned to the standards described in the NCTM documents.

Fewer mathematics educators realize that the standards documents developed by AAAS also address mathematical literacy. AAAS, a prestigious private organization of scientists, mathematicians, engineers, and educators, led the first national effort to define goals for adult science literacy in the late 1980s, with the development of *Science for All Americans* by Project 2061. Rather than a list of standards, *Science for All Americans* provides a narrative description of the interconnected web of understanding that every adult American should possess after a K–12 education, including concepts and procedures in mathematics. This narrative account reflects a strong consensus among respected scientists, mathematicians, and educators. Based on cognitive research and the expertise of teachers and teacher educators, specific mathematics learning goals were developed for the K–2, 3–5, 6–8, and 9–12 grade spans—steps along the way to achieving the vision of science literacy, which includes mathematics as its own discipline, laid out in *Science for All Americans.* These steps along the way were published in *Benchmarks*, along with rich descriptions of the context and instructional implications of those learning goals, and they were later depicted in a collection of conceptual strand maps in *Atlas of Science Literacy.*

Each of these AAAS documents includes chapters on the nature of mathematics, the mathematical world, common themes, and habits of mind, providing a rich description of the mathematical knowledge and skills all children and adults should attain. The term *science literacy* can be misleading, and many mathematics educators may assume that the only mathematics included is used to support science. Although this is a part of science literacy, the AAAS Project 2061 documents treat mathematics as both a separate discipline and as knowledge that supports and relates to science. Consider the National Science Foundation (NSF). While the name refers only to science, NSF provides significant support for mathematics (including this book). It also supports initiatives in technology and engineering. In this sense, *science* is a broad umbrella term that encompasses all of these disciplines, including mathematics. Throughout the rest of this book, we will use the term *mathematical literacy* within the broader context of *science literacy* when referring to AAAS Project 2061 resources used for CTS.

CTS provides a more valid and reliable way to interpret standards, resulting in teaching that actually reflects the intent of the standards.

Mathematics educators continue to face numerous challenges in the current standards-based teaching and learning environment, ranging from implementing new curricula to the high-stakes accountability requirements mandated by the No Child Left Behind legislation. Educators face the daunting task of applying local, state, or national standards in their own curricular, instructional, and assessment context. Local districts and states have spent considerable funds, time, and energy in developing their standards. There has been a flurry of activity in attempting to align curriculum, instruction, and assessment to standards. But there is a missing link in the chain that connects standards to efforts to implement new policies, programs, and practices. Little time has been spent helping educators interpret the content, curricular, and instructional meaning of the standards. Interpretation has been left to individual teachers. As a result, consistency and coherency—and what counts as "alignment"—vary across classrooms, districts, and states.

CTS will help educators to use standards, both national and state, and to recognize their role as central pieces in their local mathematics education system. Through CTS, teachers will develop a better understanding of standards and become personally involved in their use. Standards can then become living documents translated into classroom practice.

Cognitive Research

Research into student learning—both in general and with regard to specific ideas—is another fundamental feature of CTS. Recommendations from *How People Learn* (Bransford et al., 2000) emphasize the need to build on existing knowledge and to engage students' preconceptions—particularly when they interfere with learning. Certain preconceptions about mathematics in general, which may even be fostered in school settings, can be counterproductive to student learning. Three of these preconceptions are (1) the idea that mathematics is about learning to compute, (2) the idea that mathematics is about "following rules" to guarantee correct answers, and (3) the preconception that some people have the ability "to do" math and some don't (Donovan & Bransford, 2005). For teachers using CTS, knowledge of the conceptions about mathematics that students bring to the classroom has proven a powerful learning experience.

CTS also helps teachers to identify students' preconceptions about particular concepts and procedures. The research literature on these specific conceptions has been growing, even since the publication of *Benchmarks* and the *Principles and Standards.* However, much of the literature has been difficult for teachers to find and access. In fact, many teachers are not aware that useful summaries of the research base are available. Each CTS Guide links the mathematics topic to relevant research summaries. CTS uses two research compendia to provide concise, accessible summaries of the research that has been done around specific concepts and procedures in different curricular topics. *Benchmarks'* Chapter 15 contains research summaries linked to specific learning goals in other mathematics chapters. In Section 2 of the *Research Companion to Principles and Standards for School Mathematics* (NCTM, 2003), research findings are arranged by the five content and process standards of the *Principles and Standards* document. New research articles, linked to specific CTS Guides, are also posted and updated in a searchable database on the CTS Web site at www.curriculumtopicstudy.org.

CTS is also informed by research on adult learning. In constructivism, learners construct knowledge by modifying or rejecting existing ideas (Bransford et al., 2000). Engaging teachers in CTS by surfacing their initial ideas related to a topic, followed by a systematic study and discussion of standards and research, reflects the constructivist theory of learning. Furthermore, when teachers engage together in CTS, they create a collaborative learning environment, interacting with one another so that they can make sense of new concepts and ideas (Jonassen, 1994). Another important aspect of learning is personal reflection. Effective teacher learners use metacognitive strategies during a topic study to monitor their own ideas and thought processes, compare and contrast them with those of others, and provide reasons why they accept them (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). CTS is designed to take account of all these insights into adult learning.

> In CTS, teachers interact with the information in the standards and research and filter that information through their everyday experiences with students.

Effective Professional Development

A seminal publication for professional development in science and mathematics, *Designing Professional Development for Teachers of Science and Mathematics* (Loucks-Horsley et al., 2003) has informed CTS as a tool for transformative teacher learning. This book describes changes in mathematics education and professional development that reveal an urgent need for new teacher learning tools and materials focused on content. Some of these changes, and the way CTS responds to them, are described below:

1. The knowledge base about learning, teaching, the nature of mathematics, and professional development is growing. Our current knowledge base about teaching and learning has expanded exponentially since the rise of standards. The findings from *How People Learn* (Bransford et al., 2000) and recent papers in professional journals have increased our knowledge of how students learn specific ideas, of the misconceptions students are likely to hold, and of the developmental implications for introducing ideas in different grades. This literature reveals more about transformative learning for teachers and the importance of developing both content knowledge and pedagogical content knowledge. Consequently, teachers need tools such as CTS so that they can develop the kind of knowledge that will allow them to implement new practices successfully. CTS helps teachers access this knowledge.

2. Standards are more widely consulted as school districts shape their vision of teaching and learning. Standards are now commonplace in most schools, but implementation is still a struggle. Most state standards consulted the learning goals in national standards but also rewrote them, using performance verbs and broader descriptions of content. Like a game of telephone tag, by the time these standards reached the teachers, their clarity and specificity were lost. Consequently, many state standards leave enough ambiguity so that teachers may continue doing the same things they were doing before while claiming that they meet the standards. There is growing recognition that national mathematics standards are essential to understanding the specific intent of state standards. Through CTS, teachers can clarify the meaning and intent of their state standards, recognize the authentic changes they demand, and increase the coherency and consistency of their implementation.

3. Content and pedagogical content knowledge are playing a greater role in professional development programs. Professional development for mathematics teachers has shifted from a schoolwide focus on discipline-generic opportunities to learning experiences that are directly connected to the mathematics content they teach as well as the instructional materials they use. CTS reflects this shift by helping individuals and groups of teachers study the content and pedagogical implications of the mathematics topics they teach. "In order for teachers to demonstrate the highest levels of pedagogical content knowledge, they must have sufficient subject matter knowledge. With limited mathematical understanding, teachers' pedagogical content knowledge is restricted" (Loucks-Horsley et al., 2003, p. 40).

> Teachers who experience CTS together draw on the same knowledge base and use common language in their professional conversations about teaching and learning mathematics.

4. "Job-embedded," "practice-based," and "collegial" forms of professional development are more widely accepted, researched, and practiced. Teachers can embed CTS into their daily practice and preparation for instruction, including their preparation to implement new instructional materials and assessments. Furthermore, CTS is well-suited to collegial structures such as study groups, lesson study, and collaborative analysis of student work. By using CTS up-front, teachers socially construct their understanding and ground their professional development in a common knowledge base.

RESEARCH ON READERS' INTERACTION WITH TEXT

The CTS process involves a substantial amount of reading and analysis. This may appear to contradict the principle of active learning in professional development. An NSF *Foundations Series* research monograph, however, has examined ways in which a constructivist paradigm can facilitate teachers' learning from and with text material as part of a strategy called "Gathering and Making Sense of Information" (National Science Foundation, 2002). This strategy provides a theoretical rationale and empirical support for CTS as an effective strategy for both individual learning and learning within a community.

Selected readings can be an integral part of constructing a personal understanding of content, standards-based reform, and the use of cognitive research findings. And recent research on reading shows how CTS readings can also become part of an active and socially constructed process:

> Reading researchers have argued that reading does not need to occur as an isolated, or even individual activity. First, reading should be purposeful. In other words, teachers should read either to address questions that *they* feel the need to know more about or because their concerns could not be resolved through discussion. Reading can also be a catalyst for other experiences. Indeed, reading can fulfill many functions while teachers inquire into any topic. (Siegel, Borasi, & Fonzi, 1998). Readings can provide background information, raise questions for further inquiry about a topic, synthesize different points of view, and offer models for teachers' own practice. Reading is not a passive or straightforward matter of decoding or extracting information from text (e.g., Pearson & Fielding, 1991; Rosenblatt, 1994). Rather, readers construct meaning in interaction with the text, their own background and interests, and their purposes for reading the text. Furthermore, such construction of meaning can be even more productive when it is augmented by interactions with other learners so that different interpretations can be shared and discussed. (NSF, 2002, p. 000)

Readings in the CTS Guides reflect research studies and the collective wisdom of the hundreds of researchers, scientists, mathematicians, and educators who contributed to national standards and of mathematics trade books authored by highly respected mathematicians. Thus they are automatically grounded in research and accurate mathematics.

Reading and analyzing text in a social context led by a skilled facilitator is preferred, but CTS can also be useful as a stand-alone process for individual teachers, particularly teachers in isolated areas or those constrained by limited release time for workshops. Through CTS, those teachers can still take charge of their own professional development through a self-directed study.

MATHEMATICS TEACHERS AND TEACHING

Standards and research provide a sound theoretical foundation and vision for student and teacher learning, but the rubber meets the road in the classroom. What does it take to be effective in this new vision of mathematics teaching? What do teachers need to know and be able to do to be effective?

Mathematics content—including an understanding of central facts, principles, concepts, ideas, procedures, and important generalizations within the discipline and how they are organized—is at the heart of effective teaching, and thus student learning.

Content Knowledge

Research studies have found that high school mathematics teachers with a standard certification in their field of instruction (usually indicating coursework in both subject matter and education methods) had higher-achieving students than teachers teaching without certification in their subject area (Darling-Hammond, 2000; Monk, 1994). However, the current reality is that many classrooms lack a highly qualified teacher. Many of the nation's teachers are not adequately prepared to teach mathematics using standards-based approaches and in ways that bolster student learning and achievement (NRC, 2001). Data from *The Status of High School Mathematics Teaching* (Whittington, 2002) reveal that only 40% of secondary mathematics teachers have taken coursework in all areas recommended by NCTM: abstract algebra, geometry, calculus, data analysis and statistics, applications of mathematics/problem solving, and history of math. The data also show that although teachers reported the importance of emphasizing concepts and reasoning, the highest percentage of class time was spent solving textbook problems, reviewing homework, and practicing routine algorithms. The NRC (2001) addresses this area of concern in the publication, *Adding It Up: Helping Children Learn Mathematics:*

> The preparation of U.S. preschool to middle school teachers often falls short of equipping them with the knowledge they need for helping students develop mathematical proficiency. Many students in grades Pre-K to 8 continue to be taught by teachers who may not have appropriate certification at that grade and who have at best a shaky grasp of mathematics. (p. 4)

Teachers cannot effectively promote learning beyond their own mathematical content knowledge (Ma, 1999). However, one of the myths about the need for teachers to learn content is that merely taking a content course or having other learning experiences that focus solely on content will help teachers become better mathematics teachers. Some teachers pursue content studies for their own enjoyment through less formal venues such as immersions in a scientific or engineering setting that requires use of mathematics, solving challenging mathematics problems on their own, or attending seminars and presentations led by mathematicians. Although all these experiences are valuable and may increase teachers' content knowledge, they do not always link the content adults learn to the classroom where they teach. Content learning that is disconnected from the content taught in the classroom, including strategies for making the content accessible to learners, will not necessarily result in improved student learning. Ongoing opportunities for teacher learning—especially professional development programs that focus on content and how to teach it—coupled with tools like CTS can close the gap between what teachers know and what they need to know to teach effectively. (Loucks-Horsley et al., 2003).

Effective teaching reflects an understanding not only of the content but also of how concepts and procedures relate to and build on one another. For example, before most students are able to understand a given computational algorithm, they must have the chance to develop, explore, and explain their strategies, comparing them to others' strategies. Furthermore, developing computational fluency can both enable and be enabled by investigating data, identifying and generating patterns, and conducting explorations with shapes (NCTM, 2000). Teachers who know how the content builds from understanding and relating ideas within and across topics are better able to diagnose and address learning difficulties. They know what questions to ask when students are engaged in mathematical inquiry and which learning pathways will help students the most.

CTS is designed to help teachers identify the content they need to understand as mathematically literate adults. Two resources used in CTS that can improve teachers' adult content knowledge are *Science for All Americans* (AAAS, 1990) and *Beyond Numeracy* (Paulos, 1992). The former describes the specific mathematical ideas and skills that are important for adults who will encounter mathematics in their daily lives, including teachers of every subject area and grade level. Reading *Science for All Americans* is also helpful for teachers who already have a background or major in mathematics or science, as it describes how ideas come together in an integrated picture of science and mathematics.

Beyond Numeracy (Paulos, 1992) is a mathematics adult trade book written by a mathematician. Often when teachers do not understand a topic they need to teach, they turn to a textbook. But textbook language is stilted and technical and is often more focused on procedures. Trade books, such as *Beyond Numeracy*, explain mathematics to adults in vivid, comprehensible ways.

Teacher content knowledge is also linked to the research base on students' ideas. Providing opportunities for teachers to examine their own mathematical ideas can help them change instructional strategies that may unintentionally convey inaccurate content.

> Many of the same misconceptions and alternative ways of thinking that research has documented for K–12 students can be found in prospective and practicing mathematics teachers.

Pedagogical Content Knowledge

Teaching for understanding requires more than content knowledge:

> [Teachers] also must be skilled in helping students develop an understanding of the content, meaning that they need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding. (Weiss, Pasley, Smith, Banilower, & Heck, 2003, p. 28)

These skills constitute a teacher's specialized professional knowledge, called pedagogical content knowledge. Pedagogical content knowledge is an understanding of what makes the learning of specific topics easy or difficult for learners, and it includes knowledge of ways of representing and formulating subject matter to make it comprehensible to learners (Cochran, DeRuiter, & King, 1993; Fernández-Balboa & Stiehl, 1995; Shulman, 1986; Van Driel, Verloop, & De Vos, 1998). What is the important content, and what should children at the different grade or age levels know with respect to the content? What common misunderstandings do students have with respect to the content? Knowing the answers to these questions sets the course for making important pedagogical choices in the classroom to guide learning. A recent NRC (2001) report on K–8 mathematics emphasizes the importance of this type of knowledge:

> Effective teaching—teaching that fosters the development of mathematical proficiency over time—can take a variety of forms, each with its own possibilities and risks. All forms of instruction can best be examined from the perspective of how teachers, students, and content interact in contexts to produce teaching and learning. The effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks. Effectiveness depends on *enactment*, on the mutual and interdependent interaction of the three elements—mathematical content, teacher, students—as instruction unfolds. (p. 9)

This is why CTS is called the "missing link." It helps teachers think about how to create meaningful and appropriate learning opportunities for students. Examining the essays in *Benchmarks* and *Principles and Standards* and using the research summaries help teachers identify topic-specific strategies that support student learning in mathematics. These strategies can include use of particular representations, tools, and investigative activities that directly challenge students' preconceptions. Such strategies also include knowing how to sequence those experiences in such a way as to scaffold students' developing understanding. In a similar vein, *Atlas of Science Literacy* provides a graphic representation for teachers to examine students' growth of understanding as ideas and skills begin in K–2 grades and become increasingly sophisticated by the end of high school. It also provides a way for teachers to think about the connections within and between the various topics they teach in mathematics as well as the connections to science.

Beliefs About Teaching and Learning

Research documents the impact of teachers' beliefs about mathematics on their curricular decisions and instructional practices. Teacher beliefs have a strong impact on their understanding of the nature of mathematics as a discipline; what constitutes legitimate mathematical procedures, results, and justifications; and what constitutes desirable goals and acceptable outcomes for K–12 mathematics instruction (Thompson, 1992).

> Most teachers, regardless of whether they are generalists or specialists, never had the opportunity to make their beliefs explicit in traditional teacher preparation. Readings and discussions about the discipline of mathematics are notably absent from school mathematics and even college level courses. Nevertheless, because they studied in traditional mathematics classes, most teachers hold deep-seated beliefs that mathematics is a body of absolute truths with little room for creativity or personal judgment. This means that, as teachers, they are likely to value correct answers over tentative conjectures, standard procedures over personal approaches to solutions, and facts and algorithms over inductive problem solving and reasoning skills. (NSF, 2002, p. 13)

> New materials and strategies for teacher learning must provide opportunities for teachers to critically examine their personal beliefs about mathematics teaching and learning and offer ways to help them develop new beliefs grounded in a reform vision.

The common teacher beliefs described above conflict with the current vision of mathematics teaching and learning reflected in the standards. CTS provides a mechanism to confront and alter existing beliefs. The CTS approach draws out teachers' knowledge and beliefs about a topic and how students learn it, and it helps them connect new ideas with their previous ideas and beliefs.

One way teachers change or reinforce their beliefs is through discourse in a social setting. Having an opportunity to present one's own ideas after studying a topic and hearing and reflecting on the ideas of others is an empowering experience. CTS encourages discourse with others that facilitates meaning-making and leads to changes in long-held beliefs. This kind of learning is highly personal. Perhaps most important, it can resolve the dissonance between long-held beliefs and new thinking, resulting in changed practice (Mezirow, 1997).

The cognitive research on how people learn has also begun to influence beliefs about teaching and learning. Mathematics teachers who have used CTS display a significant shift in how they perceive their own role. More and more, teachers are moving away from seeing teaching as telling to seeing it as facilitating learning.

They are moving from the idea that only some children can learn to the belief that all children can learn challenging content, a central premise of the standards movement.

Having a Professional Knowledge Base

A teacher's role as facilitator of learning for all students is complex and demanding (Loucks-Horsley et al., 2003). Teaching is enhanced when teachers operate from a body of specialized professional knowledge that is based on research about how to teach and how people learn (Stigler & Hiebert, 1999). In the section of *Principles and Standards of School Mathematics* (NCTM, 2000) titled, Working Together to Achieve the Vision, the use of current research is included as an important component of improving mathematics teaching and learning:

> [Teachers] must adjust their practices and extend their knowledge to reflect changing curricula and technologies and to incorporate new knowledge about how students learn mathematics. . . . Teachers must develop their own professional knowledge using research, the knowledge base of the profession, and their own experiences as resources. (p. 370)

CTS helps teachers develop such a professional knowledge base. Teachers who use CTS can describe and support their choices of instructional strategies by citing the research or national standards. Conversations among teachers who have used CTS become more deliberate and grounded in a common body of professional knowledge. For example, in observations of teachers during the CTS process, they were frequently heard to say, "According to the research it is common for students to have trouble with this concept. What can we do differently to help them?" As they engaged in conversations about what is important to teach, they would say, "Let's look at the research and standards." This language reflects the fact that those teachers were making evidence-based decisions that move them beyond their personal opinions while still respecting practitioner wisdom and experience.

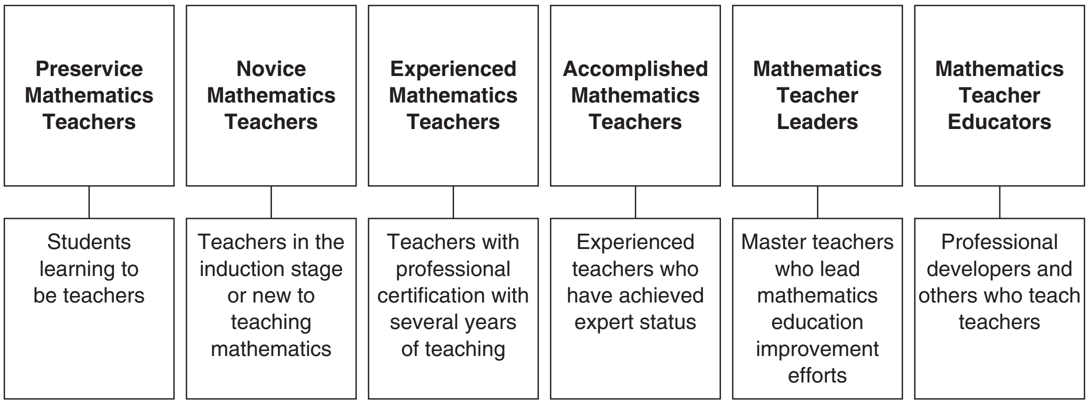
THE TEACHER PROFESSIONAL CONTINUUM

Traditionally, preservice and inservice mathematics teacher education was expected to produce and support teachers competent in the knowledge and skills they would need to sustain their learning over the period of their mathematics teaching career. However, with new legislation regarding teacher quality, standards-based reform of mathematics teaching and learning, and the diverse knowledge and experience of teachers, it is now recognized that teachers' professional development needs to be differentiated and continuous—building from their undergraduate education into a lifelong career of learning.

Learning to teach can be regarded as a continuum of professional experiences (Bransford et al., 2000). Research on professional development is linked to this concept of a professional continuum (see Figure 1.6). This continuum begins with teachers' experiences as they progress through their K–12 education,

As teacher characteristics change, their needs for professional development change accordingly.

Figure 1.6 The CTS Mathematics Teaching Continuum



and it builds to include preservice programs, induction, professional development for experienced teachers, and professional leadership activities. Studies show that teachers develop differently and have different attitudes, knowledge, skills, and behaviors at various points during their career. Their concerns about teaching, their instructional behaviors, their mathematical content knowledge, their understanding of how students learn mathematics, and their perceptions of themselves, their work, and their profession all change over time. Teachers with different levels of expertise use and engage in CTS in different ways that are meaningful to them. In addition, CTS provides a process and set of tools that teachers can use to lead professional development and support the learning of their less experienced colleagues.

NSF's Teacher Professional Continuum Program, which provided funding for the development of CTS, is designed to provide support for a comprehensive, coherent, and integrated sequence of lifelong learning for teachers (NSF, 2002). In response, CTS is designed to consider teachers' professional development, inservice, and professional growth opportunities in light of their career stage. CTS supports personalized learning targeted toward the unique needs of an individual teacher as well as members of a learning community and moves them along a professional learning pathway. CTS is a tool designed to address continuous learning, both inside and outside of the classroom, by helping educators at all career stages evolve in the mathematics teaching profession.