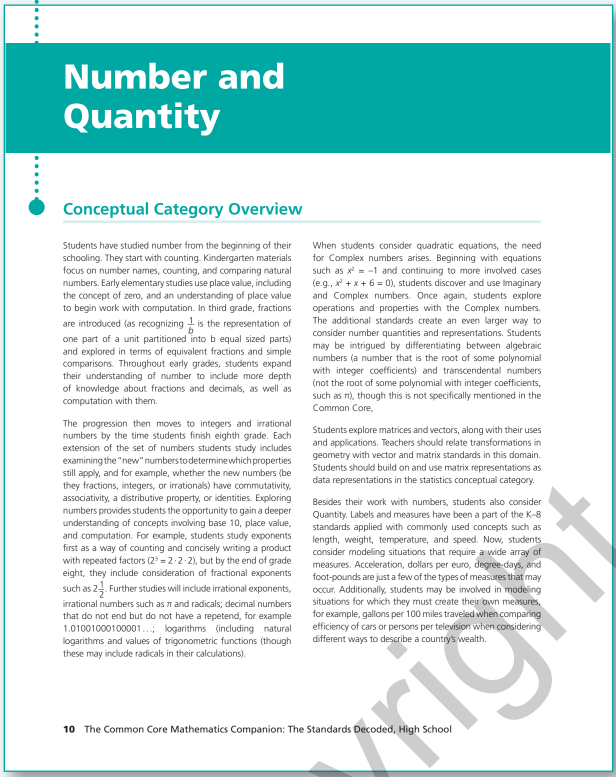
The Common Core Companion at a Glance

Conceptual Category Overview: Gives a brief description of the conceptual categories or strands of mathematics, allowing you to see the big picture of what students should learn across the high school grades.

Direct Connections: Explains connections to standards from the middle grades.



Number and Quantity

Conceptual Category Overview

Students have studied number from the beginning of their schooling. They start with counting. Kindergarten materials focus on number names, counting, and comparing natural numbers. Early elementary studies use place value, including the concept of zero, and an understanding of place value to begin work with computation. In third grade, fractions are introduced (as recognizing $\frac{1}{b}$ is the representation of one part of a unit partitioned into b equal sized parts) and explored in terms of equivalent fractions and simple comparisons. Throughout early grades, students expand their understanding of number to include more depth of knowledge about fractions and decimals, as well as computation with them.

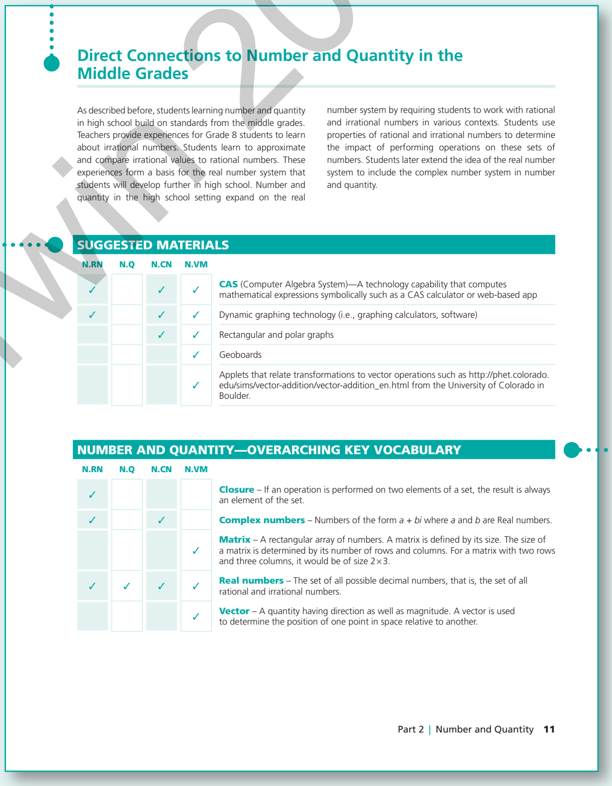
The progression then moves to integers and irrational numbers by the time students finish eighth grade. Each extension of the set of numbers students study includes examining the "new" numbers to determine which properties still apply, and for example, whether the new numbers (be they fractions, integers, or irrationals) have commutativity, associativity, a distributive property, or identities. Exploring numbers provides students the opportunity to gain a deeper understanding of concepts involving base 10, place value, and computation. For example, students study exponents first as a way of counting and concisely writing a product with repeated factors ($2^3 = 2 \cdot 2 \cdot 2$), but by the end of grade eight, they include consideration of fractional exponents such as $2\frac{1}{2}$. Further studies will include irrational exponents, irrational numbers such as π and radicals; decimal numbers that do not end but do not have a repetend, for example 1.01001000100001 . . .; logarithms (including natural logarithms and values of trigonometric functions (though these may include radicals in their calculations).

When students consider quadratic equations, the need for Complex numbers arises. Beginning with equations such as $x^2 = -1$ and continuing to more involved cases (e.g., $x^2 + x + 6 = 0$), students discover and use Imaginary and Complex numbers. Once again, students explore operations and properties with the Complex numbers. The additional standards create an even larger way to consider number quantities and representations. Students may be intrigued by differentiating between algebraic numbers (a number that is the root of some polynomial with integer coefficients) and transcendental numbers (not the root of some polynomial with integer coefficients, such as π), though this is not specifically mentioned in the Common Core,

Students explore matrices and vectors, along with their uses and applications. Teachers should relate transformations in geometry with vector and matrix standards in this domain. Students should build on and use matrix representations as data representations in the statistics conceptual category.

Besides their work with numbers, students also consider Quantity. Labels and measures have been a part of the K–8 standards applied with commonly used concepts such as length, weight, temperature, and speed. Now, students consider modeling situations that require a wide array of measures. Acceleration, dollars per euro, degree-days, and foot-pounds are just a few of the types of measures that may occur. Additionally, students may be involved in modeling situations for which they must create their own measures, for example, gallons per 100 miles traveled when comparing efficiency of cars or persons per television when considering different ways to describe a country's wealth.

10 The Common Core Mathematics Companion: The Standards Decoded, High School



Direct Connections to Number and Quantity in the Middle Grades

As described before, students learning number and quantity in high school build on standards from the middle grades. Teachers provide experiences for Grade 8 students to learn about irrational numbers. Students learn to approximate and compare irrational values to rational numbers. These experiences form a basis for the real number system that students will develop further in high school. Number and quantity in the high school setting expand on the real number system by requiring students to work with rational and irrational numbers in various contexts. Students use properties of rational and irrational numbers to determine the impact of performing operations on these sets of numbers. Students later extend the idea of the real number system to include the complex number system in number and quantity.

SUGGESTED MATERIALS

N.RN	N.Q	N.CN	N.VM	
✓		✓	✓	**CAS** (Computer Algebra System)—A technology capability that computes mathematical expressions symbolically such as a CAS calculator or web-based app
✓		✓	✓	Dynamic graphing technology (i.e., graphing calculators, software)
		✓	✓	Rectangular and polar graphs
			✓	Geoboards
			✓	Applets that relate transformations to vector operations such as http://phet.colorado.edu/sims/vector-addition/vector-addition_en.html from the University of Colorado in Boulder.

NUMBER AND QUANTITY—OVERARCHING KEY VOCABULARY

N.RN	N.Q	N.CN	N.VM	
✓				**Closure** – If an operation is performed on two elements of a set, the result is always an element of the set.
✓		✓		**Complex numbers** – Numbers of the form $a + bi$ where a and b are Real numbers.
			✓	**Matrix** – A rectangular array of numbers. A matrix is defined by its size. The size of a matrix is determined by its number of rows and columns. For a matrix with two rows and three columns, it would be of size 2×3.
✓	✓	✓	✓	**Real numbers** – The set of all possible decimal numbers, that is, the set of all rational and irrational numbers.
			✓	**Vector** – A quantity having direction as well as magnitude. A vector is used to determine the position of one point in space relative to another.

Part 2 | Number and Quantity **11**

Suggested Materials: Provides teachers with a list of materials that will be helpful in introducing the ideas within the domains that follow.

Key Vocabulary: Vocabulary included in the conceptual category. This terminology can be used for building a word wall in the classroom.

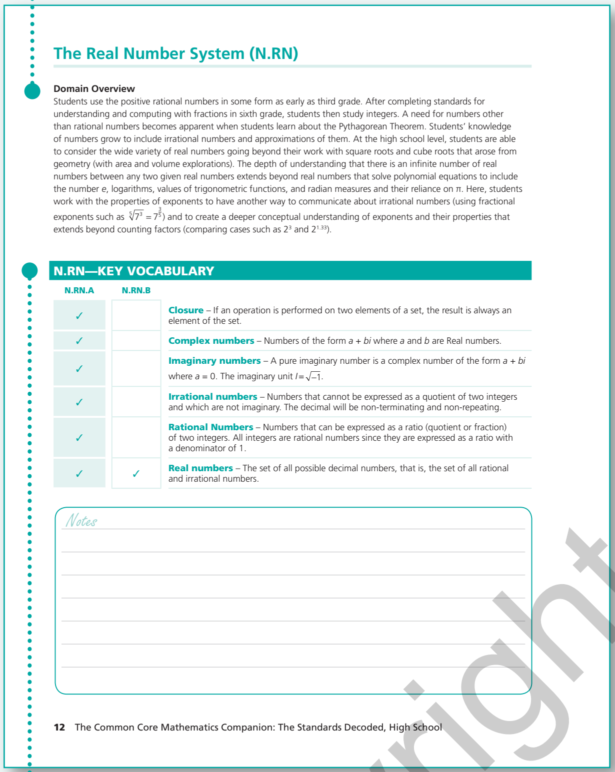
Identifying information for this cluster, stated as: Conceptual Category. Domain. Cluster.

Cluster: Statements that summarize groups of related standards. Note that standards from different clusters may sometimes be closely related.

Standards: Mathematical statements that define what students should understand and be able to do.

Conceptual Category and Domain: Focus for this group of standards.

Domain Overview: Gives a brief description of the big ideas covered in each domain.



The Real Number System (N.RN)

Domain Overview

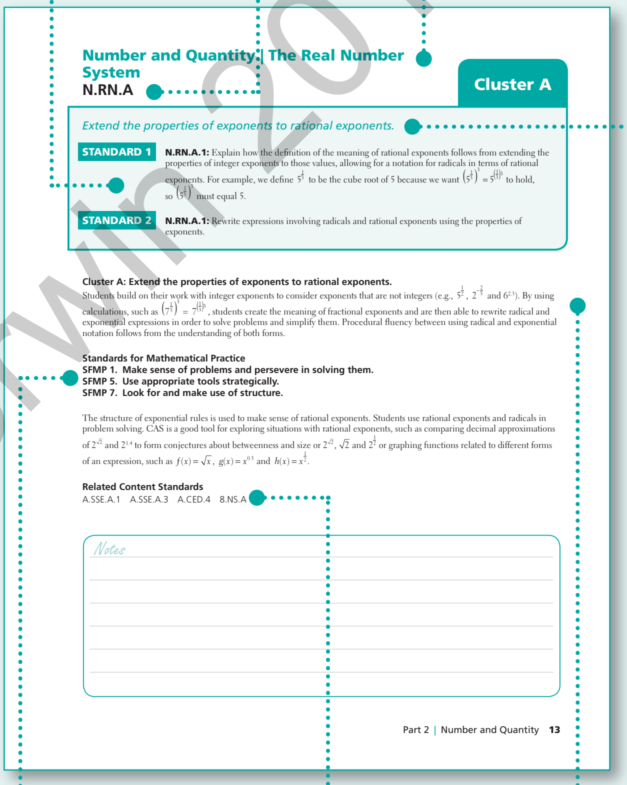
Students use the positive rational numbers in some form as early as third grade. After completing standards for understanding and computing with fractions in sixth grade, students then study integers. A need for numbers other than rational numbers becomes apparent when students learn about the Pythagorean Theorem. Students' knowledge of numbers grow to include irrational numbers and approximations of them. At the high school level, students are able to consider the wide variety of real numbers going beyond their work with square roots and cube roots that arose from geometry (with area and volume explorations). The depth of understanding that there is an infinite number of real numbers between any two given real numbers extends beyond real numbers that solve polynomial equations to include the number e, logarithms, values of trigonometric functions, and radian measures and their reliance on π. Here, students work with the properties of exponents to have another way to communicate about irrational numbers (using fractional exponents such as $\sqrt[5]{7^3} = 7^{\frac{3}{5}}$) and to create a deeper conceptual understanding of exponents and their properties that extends beyond counting factors (comparing cases such as 2^3 and $2^{1.33}$).

N.RN—KEY VOCABULARY

N.RN.A	N.RN.B	
✓		**Closure** – If an operation is performed on two elements of a set, the result is always an element of the set.
✓		**Complex numbers** – Numbers of the form $a + bi$ where a and b are Real numbers.
✓		**Imaginary numbers** – A pure imaginary number is a complex number of the form $a + bi$ where $a = 0$. The imaginary unit $I = \sqrt{-1}$.
✓		**Irrational numbers** – Numbers that cannot be expressed as a quotient of two integers and which are not imaginary. The decimal will be non-terminating and non-repeating.
✓		**Rational Numbers** – Numbers that can be expressed as a ratio (quotient or fraction) of two integers. All integers are rational numbers since they are expressed as a ratio with a denominator of 1.
✓	✓	**Real numbers** – The set of all possible decimal numbers, that is, the set of all rational and irrational numbers.

Notes

12 The Common Core Mathematics Companion: The Standards Decoded, High School



Number and Quantity | The Real Number System
N.RN.A

Cluster A

Extend the properties of exponents to rational exponents.

STANDARD 1 **N.RN.A.1:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)3}$ to hold, so $\left(5^{\frac{1}{3}}\right)^3$ must equal 5.

STANDARD 2 **N.RN.A.1:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Cluster A: Extend the properties of exponents to rational exponents.
Students build on their work with integer exponents to consider exponents that are not integers (e.g., $5^{\frac{1}{2}}$, $2^{-\frac{2}{3}}$ and $6^{2.3}$). By using calculations, such as $\left(7^{\frac{1}{3}}\right)^3 = 7^{\left(\frac{1}{3}\right)3}$, students create the meaning of fractional exponents and are then able to rewrite radical and exponential expressions in order to solve problems and simplify them. Procedural fluency between using radical and exponential notation follows from the understanding of both forms.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 5. Use appropriate tools strategically.
SFMP 7. Look for and make use of structure.

The structure of exponential rules is used to make sense of rational exponents. Students use rational exponents and radicals in problem solving. CAS is a good tool for exploring situations with rational exponents, such as comparing decimal approximations of $2^{\sqrt{2}}$ and $2^{1.4}$ to form conjectures about betweenness and size or $2^{\sqrt{2}}$, $\sqrt{2}$ and $2^{\frac{1}{2}}$ or graphing functions related to different forms of an expression, such as $f(x) = \sqrt{x}$, $g(x) = x^{0.5}$ and $h(x) = x^{\frac{1}{2}}$.

Related Content Standards
A.SSE.A.1 A.SSE.A.3 A.CED.4 8.NS.A

Notes

Part 2 | Number and Quantity 13

Key Vocabulary: Highlights the specific vocabulary relevant to each domain. Students should be able to use these terms in talking about mathematics; standard for Mathematical Practice 6: Attend to Precision calls for students to use mathematical terminology appropriately.

Standards for Mathematical Practice: Although it is likely you will use a variety of Standards for Mathematical Practice in teaching each cluster, this section gives examples of how you might incorporate some of the practices into your instruction on this topic.

Each cluster begins with a brief description of the mathematics in that cluster.

Related Content Standards: Provides a list of standards connected to this topic, including those at other grade levels and conceptual categories. Consider the related standards as you plan instruction for each cluster.

What the TEACHER does: An overview of actions the teacher might take in introducing and teaching the standard. This is not meant to be all-inclusive, but rather to give you an idea of what classroom instruction might look like. Illustrations may be included, detailing how to use materials to teach a concept when using models and representations called for in the standard.

Standard: The standard as written in the Common Core, followed by an explanation of the meaning of the mathematics in that standard, including examples.

What the STUDENTS do: Some examples of what students may do as they explore and begin to understand the standard. This is not intended to be directive, but rather to frame what student actions may look like.

STANDARD 2 (N.RN.A.2)

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Students are able to use both radical and exponential forms to write expressions and can translate flexibly between them. Students use symbolic examples, such as $a^2\sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{\frac{5}{2}}$, and contextual examples, like solving $V = \frac{4}{3}\pi r^3$ for r.

What the TEACHER does:

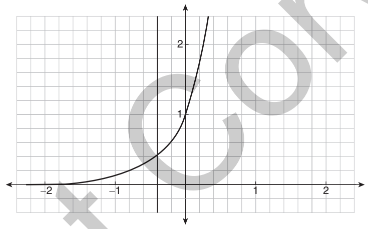
- Uses problems that allow students to use either radical or exponential forms and requires them to explain their reasoning for their choice.
- Solves contextual problems such as solving the volume of a cube for one side, $V = s^3$, $s = V^{\frac{1}{3}} = \sqrt[3]{V}$
- Requires students to discuss the meanings of their computations when rewriting and simplifying radical and rational exponent expressions.

What the STUDENTS do:

- Explain the meaning of rational exponents in terms of radicals and roots.
- Translate fluently between radical and exponential forms.
- Explain their reasoning when using either notation to solve problems involving radicals.

Addressing Student Misconceptions and Common Errors

Negative exponents can be a problem when using fractional exponents. Students often think $9^{-\frac{1}{2}}$ means –3 instead of $\frac{1}{3}$. Using a calculator to calculate $9^{\frac{-1}{2}}$ helps, as does looking at the graph of $y = 9^x$ and $x = -\frac{1}{2}$, to see where the functional value occurs.



The curve is $y = 9^x$, and the vertical line is $x = -\frac{1}{2}$. The scale shows the intersection of the curve and graph is a positive number that is between zero and 0.4, so –3 is excluded as a solution while $\frac{1}{3}$ appears as a viable estimate of the intersection value.

Connections to Modeling

Solving problems that involve formulas with exponents and/or radicals. Solving problems that involve volume and area.

Related Content Standards

A.SSE.B.3 F.IF.C.7e

Connections to Modeling: Modeling is signaled out at the high school level as a curricular goal in and of itself, and it is a unifying theme across all conceptual categories. This section provides suggestions for integrating modeling into classroom instruction.

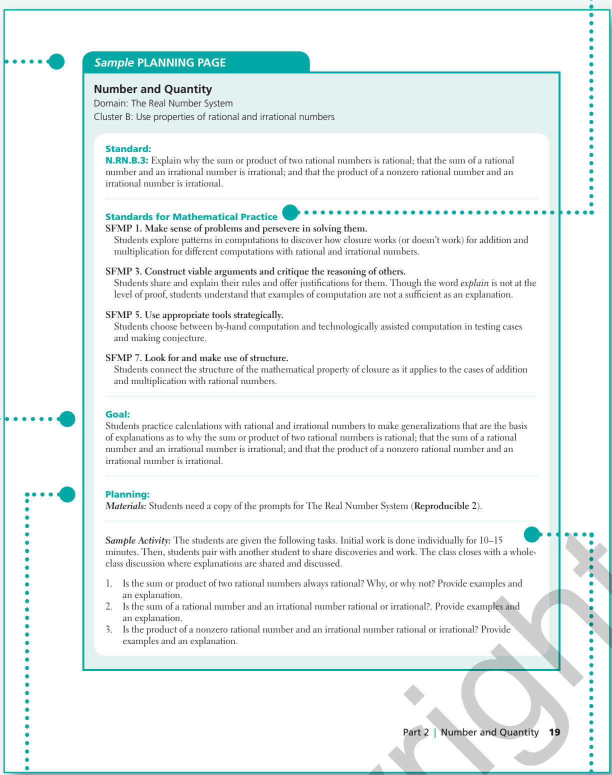
Addressing Student Misconceptions and Common Errors: Each standard includes a misconception or common student error around the standard and suggested actions to address those misconceptions or errors.

Standards for Mathematical Practice: The Mathematical Practices emphasized in this sample plan are included.

Differentiating Instruction: Suggestions to address the need of struggling learners along with extension ideas to challenge other students are included here.

Sample Planning Page: At the end of each domain, you will find a sample planning page based on one standard or group of standards for that domain. While these are not complete lesson plans, they provide ideas, activities, and a structure for planning.

Questions/Prompts: This section provides questions or prompts you may use to help build student understanding and encourage student thinking.



Sample PLANNING PAGE

Number and Quantity
Domain: The Real Number System
Cluster B: Use properties of rational and irrational numbers

Standard:
N.RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
Students explore patterns in computations to discover how closure works (or doesn't work) for addition and multiplication for different computations with rational and irrational numbers.

SFMP 3. Construct viable arguments and critique the reasoning of others.
Students share and explain their rules and offer justifications for them. Though the word *explain* is not at the level of proof, students understand that examples of computation are not a sufficient as an explanation.

SFMP 5. Use appropriate tools strategically.
Students choose between by-hand computation and technologically assisted computation in testing cases and making conjecture.

SFMP 7. Look for and make use of structure.
Students connect the structure of the mathematical property of closure as it applies to the cases of addition and multiplication with rational numbers.

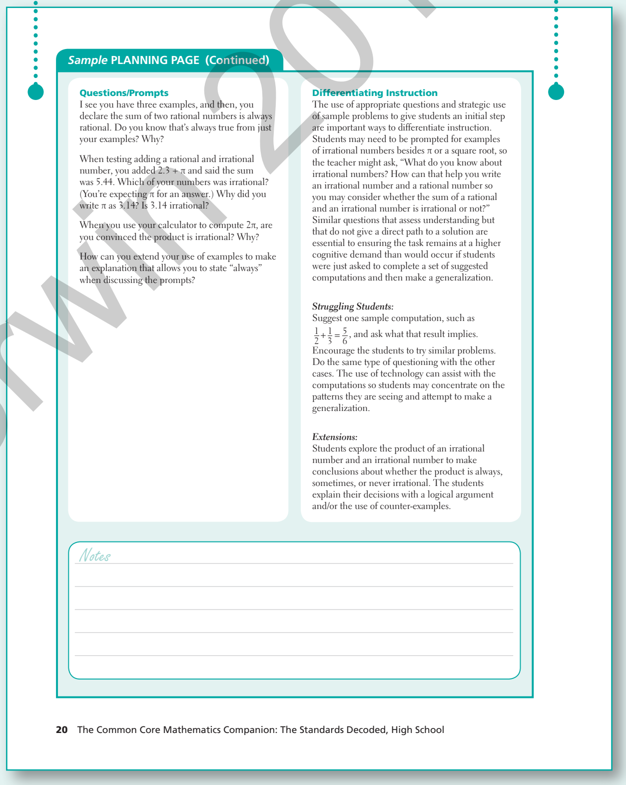
Goal:
Students practice calculations with rational and irrational numbers to make generalizations that are the basis of explanations as to why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Planning:
Materials: Students need a copy of the prompts for The Real Number System (**Reproducible 2**).

Sample Activity: The students are given the following tasks. Initial work is done individually for 10–15 minutes. Then, students pair with another student to share discoveries and work. The class closes with a whole-class discussion where explanations are shared and discussed.

1. Is the sum or product of two rational numbers always rational? Why, or why not? Provide examples and an explanation.
2. Is the sum of a rational number and an irrational number rational or irrational?. Provide examples and an explanation.
3. Is the product of a nonzero rational number and an irrational number rational or irrational? Provide examples and an explanation.

Part 2 | Number and Quantity 19



Sample PLANNING PAGE (Continued)

Questions/Prompts
I see you have three examples, and then, you declare the sum of two rational numbers is always rational. Do you know that's always true from just your examples? Why?

When testing adding a rational and irrational number, you added $2.3 + \pi$ and said the sum was 5.44. Which of your numbers was irrational? (You're expecting π for an answer.) Why did you write π as 3.14? Is 3.14 irrational?

When you use your calculator to compute 2π, are you convinced the product is irrational? Why?

How can you extend your use of examples to make an explanation that allows you to state "always" when discussing the prompts?

Differentiating Instruction
The use of appropriate questions and strategic use of sample problems to give students an initial step are important ways to differentiate instruction. Students may need to be prompted for examples of irrational numbers besides π or a square root, so the teacher might ask, "What do you know about irrational numbers? How can that help you write an irrational number and a rational number so you may consider whether the sum of a rational and an irrational number is irrational or not?" Similar questions that assess understanding but that do not give a direct path to a solution are essential to ensuring the task remains at a higher cognitive demand than would occur if students were just asked to complete a set of suggested computations and then make a generalization.

Struggling Students:
Suggest one sample computation, such as $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, and ask what that result implies. Encourage the students to try similar problems. Do the same type of questioning with the other cases. The use of technology can assist with the computations so students may concentrate on the patterns they are seeing and attempt to make a generalization.

Extensions:
Students explore the product of an irrational number and an irrational number to make conclusions about whether the product is always, sometimes, or never irrational. The students explain their decisions with a logical argument and/or the use of counter-examples.

Notes

20 The Common Core Mathematics Companion: The Standards Decoded, High School

Materials: The materials used in the Sample Activity are listed.

Sample Activity: An example of an activity that addresses this standard is provided.

Goal: The purpose of this activity and how it connects to previous and future ideas is stated.