# **Chapter 1**

## "THE WIZARD'S TALE"

## Doris Lawson Tells Fourth Graders an Oral Story

his chapter provides an example of how a fourth grade teacher, Doris Lawson, uses oral storytelling to present an epic mathematics tale.

Doris Lawson describes herself as a regular teacher—that is, before she changed the way she teaches mathematics. Doris used to follow her mathematics textbook. She would present her fourth graders the lesson in their textbook, give them practice problems, and then assign homework. Homework was checked at the beginning of every math lesson, and a test was given at the end of each week.

About 1990, after teaching this way for 10 years, Doris decided to reinvent the way she taught in order to make teaching more enriching for both her students and herself. First, Doris started using manipulatives. She used base ten blocks, geoboards, pattern blocks, and fraction bars to help give meaning to her lessons. Later she discovered math games: social games with names like addition war, division bingo, and multiplication dominoes. Doris created twenty to thirty math games that reinforced and extended the ideas presented in each chapter of her text-book using materials such as egg cartons, poster board, tongue depressors, and wood cubes.

By 1992 Doris discovered children's literature and began using mathematical stories with her class. She felt, however, that few children's storybooks developed the mathematical skills that she wanted her students to learn and that when she read a book to her students they were outsiders looking in on the world of others. She wanted to get her class more involved in mathematical stories—involved in ways that deeply stimulated their fantasies and more fully developed the mathematical skills that she wanted them to learn.

As a result, I began writing mathematical oral stories for Doris, and Doris began to develop her ability to tell those stories. We began to learn how to weave fantasy tales around mathematical topics in such a way that Doris's students could listen in to the mathematical thoughts of the characters in the stories and participate in the stories by helping those characters overcome mathematical obstacles and challenges. The stories gradually became

epics—stories that lasted for many days—that guided students through several stages of learning in order to help them develop mathematical skills.

One of the stories that I wrote for Doris is "The Wizard's Tale." It is designed to help children understand the multidigit addition algorithm and acquire the skills needed for adding multidigit numbers. Doris told the story four times to four classes between 1993 and 1997 before moving from teaching fourth graders all subjects to teaching sixth, seventh, and eighth grade mathematics. Other teachers have used "The Wizard's Tale" with second, third, and fourth graders.

Let us now listen to Doris as she tells "The Wizard's Tale." It takes her 5 days to tell the story. In what follows, indented text indicates what Doris actually says while telling the story. Italicized type identifies a comment about what Doris is saying or doing. Type that is not indented describes what is occurring in Doris's classroom. The accompanying CD provides a full description of what occurred in Doris's classroom.

## DAY 1

This first session of "The Wizard's Tale" takes Doris about an hour and a half to tell. She tells it during two of her scheduled periods: mathematics and language arts. Doris announces the story by stating rather matter-of-factly, "OK, you guys, settle down. I want to tell you a story." She waits until the class is quiet and then begins. As she speaks she moves about the class gesturing with her arms and changing the tone of her voice to accentuate what she is saying.

Once upon a time there lived two wizards named Gandalf and Tinkerbell, who had one exciting adventure after another. In fact, they are still alive, are still friends, and still have awesome adventures.

Gandalf is an old man who has been a wizard for many years. Some say he is several thousand years old. Some say his story was first told by Tolkien in *The Lord of the Rings*. Tinkerbell is a fairy who is 11 years old and who has only been a wizard for 5 years. Some say her mother's story was told in *Peter Pan*. Both Gandalf and Tinkerbell are the type of wizards who can do magic. Gandalf is about 6 feet tall, has a long white beard, gray eyes, and wears old, baggy, gray clothes and a conical wizard's cap. [*Doris says this while speaking in a deep male voice*.] Tinkerbell is four inches tall, has long, straight hair, has a ring of stars that float above her head like a halo, and wears dazzling clothes with stars, lace, sparkles, and rainbows on them. [*Doris says this while speaking in a high female voice*.] Both Gandalf and Tinkerbell have the ability to change their sizes and forms. Tinkerbell can make herself as big as a giant. Gandalf can make himself into an ant and crawl through small spaces. [*Doris gestures to portray large and tiny*.]

One day Tinkerbell is daydreaming in a hammock next to her home, an enormous, old oak tree. All of a sudden, there is a big boom over Tinkerbell's head. It is so loud that Tinkerbell jumps out of her hammock and hides under a leaf. Then fireworks start to go off over Tinkerbell's head, big red and blue and yellow flowers and fountains. Tinkerbell knows what is happening now, for this is the way Gandalf communicates with her when he is in

trouble. Suddenly words appear among the fireworks, in bright red letters. They say "Help, I am trapped in Thoughtful Mountain. Come . . ." Then the fireworks and the message end abruptly.

Tinkerbell has heard of Thoughtful Mountain. It is a place where wizards and witches sometimes go to test their magic powers. It is alive and magical and poses problems, riddles, and puzzles to visitors. Many a wizard and witch have disappeared forever upon entering Thoughtful Mountain. Tinkerbell has never been there and does not want to go, but Gandalf is in trouble in Thoughtful Mountain.

So Tinkerbell runs into her tree house and then into the room that holds all of her magic powders and devices. Hurriedly she takes out her crystal ball so she can see what is happening to Gandalf, puts it on a table, claps her hands three times, utters magic words to the crystal ball, and claps her hands three more times.

Tinkerbell always claps her hands three times before and after saying magic words to make her magic work. You, from right here in our classroom, can help Tinkerbell make her magic more powerful by clapping your hands in unison three times before and after she says magic words. I will signal you when you should clap by moving my hands in front of me, like this, to indicate a silent clap.

Doris now has her class practice clapping in unison three times after she gestures a silent clap. After two tries her class can clap loudly in unison three times. She then has them help Tinkerbell activate her crystal ball by clapping in unison before and after she says the magic words, "Gamble, grumble, groumble Gandalf."

Saying those magic words and clapping make the crystal ball show what is happening to Gandalf. It shows him being slowly turned into stone inside a dark cavern. Suddenly the ball's image disappears. This means that someone or something magical—probably Thoughtful Mountain—has turned off the crystal ball's seeing power.

Tinkerbell is worried. She quickly fills a small pouch with magic devices and runs outside her house. She gets ready to do magic. [Doris signals for magic clapping, and her class responds.] "Tiba, diba, riba." [Doris signals and her class produces more magic clapping.] This turns Tinkerbell into a giant red hawk, one of the fastest flying birds in the world. And into the air Tinkerbell jumps, and off she flies toward Thoughtful Mountain [Doris flaps her arms as though they were wings as she continues.]

Tinkerbell flies for 5 hours to get to Thoughtful Mountain. As she flies she thinks of everything she knows about the mountain. It isn't much. But Tinkerbell knows where the entrance is and the magic words that will allow her to enter the mountain. She also knows that the mountain has many caverns within it, including a history cavern, a mathematics cavern, and a science cavern.

When Tinkerbell arrives at the mountain she flies to its entrance and prepares to turn herself from a red hawk back into Tinkerbell. [Doris signals

and magic clapping occurs.] "Tiba, diba, riba." [more magic clapping] Suddenly Tinkerbell looks like herself again.

Tinkerbell now gets ready to do the magic that will allow her to enter Thoughtful Mountain. [Doris signals and magic clapping occurs.] "Double fuffle, guffle, truffle." [Doris signals and more magic clapping.]

There is a great creaking sound from the mountain and a small door, just Tinkerbell's size, appears and opens. Above it are written the words in very small print: "DO NOT ENTER, unless you can answer my questions. Incorrect answers will turn you into stone."

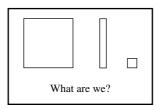
Cautiously Tinkerbell climbs up the mountain and walks through the small door. The door slams shut behind her and she is in complete darkness.

Tinkerbell takes a deep breath and gets ready to do magic. [Doris signals and magic clapping occurs.] "Twinkle, twankle, twinkle." [more magic clapping] A halo of stars begins to glow above Tinkerbell's head so brightly that she can see all around her. In front of her is a great cavern with many tunnels leading out of it. This is no ordinary cavern. One wall of the cavern, which seems to be made out of polished marble, is covered with a beautiful picture of a flower garden. The garden is constructed out of small bits of gold, diamond, rubies, sapphires, and emeralds that all seem to grow out of the wall. Tinkerbell can hardly believe her eyes, particularly when several silver butterflies fly across the wall.

But Tinkerbell did not come to Thoughtful Mountain to look at beautiful stones. She came to find her friend Gandalf. She looks at the many tunnels leading out of the main cavern and wonders aloud, "Which tunnel did Gandalf go down?" She takes her magic compass out of her pouch and tells it to find where Gandalf had walked. The magic compass is like a bloodhound tracking dog. Its needle turns around and around until it finds the scent of Gandalf's footprints. Then it points in the direction that Gandalf walked. Tinkerbell follows the magic compass and goes into one of the tunnels. She walks down the tunnel for an hour. Then the tunnel ends.

There is nothing but a wall in front of her. But her magic compass says that Gandalf walked through the wall. Tinkerbell studies the wall and sees some markings engraved on it. This is what they look like. [Doris draws this on the chalkboard.]

Exhibit 1.1



"What do the marks mean?" Tinkerbell wonders. She thinks and thinks, but can think of nothing.

Doris tells her students that Tinkerbell needs their help to figure out what the marks mean. She puts her students into groups of two and asks them to discuss in their small groups what they think the marks mean. Doris has previously introduced her class to base ten blocks, and they have many ideas. After a few minutes of class discussion Doris continues the story.

After thinking for the longest time, and not coming up with any ideas, Tinkerbell takes out her magic slate. The slate usually answers any questions that she has. It is sort of like a magical encyclopedia. Tinkerbell draws a picture of the wall markings on the magic slate and prepares to do magic. [Doris signals and magic clapping occurs.] "Sedle, seeelle, see. What are thee?" [more magic clapping] Tinkerbell watches the slate as it searches for an answer.

To involve her students in helping the story progress, Doris asks them to telepathically send their answers to Tinkerbell. She signals for her class to clap three times. Students then concentrate while they telepathically send the answer to Tinkerbell. Doris then signals for her class to clap three more times to complete the magic transmission.

As soon as Tinkerbell's magic slate receives our telepathic thoughts, it responds in a metallic, squeaky voice [that Doris imitates], "We are base ten number blocks. We are used in math. Our names are hundred, ten, and one. Ten ones equal a ten. Ten tens equal one hundred."

Now Tinkerbell remembers. Tinkerbell announces to the wall, "You are hundred, ten, and one. If ten ones are put together, they make a ten. If ten tens are put together, they make a hundred."

A great groaning echoes through the cavern as the wall at the end of the tunnel gradually melts away into nothing. On the other side of where the wall used to be is another cavern.

Tinkerbell slowly walks into the new cavern. Inside Tinkerbell sees something. It is Gandalf, three other wizards, and a witch, all of them turned to stone and frozen into the cavern's wall. Tinkerbell stands still and quietly looks around her.

On the floor not far away is a magic place value chart that looks like this. [Doris holds up a 2- by 3-foot (0.6- by 0.9-meter) piece of cardboard with a place value chart drawn on it.]

Exhibit 1.2

| hundreds | tens | ones |
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All of a sudden, a deep, booming voice says [and Doris delivers this in a deep, booming voice], "Hello! So you have come to test your thoughtfulness

against me, the Thoughtful Mountain. Well, prepare yourself! Solve my problems, or you will become stone! Here is your only hint: 'Talking Bulldozer adds.'"

Suddenly the room starts to glow pink, the shape on the floor begins to glow red, and from two different parts of the ceiling, orange pieces of stone fall to the floor. This is what the two sets of orange stone pieces look like. [Doris places these configurations of base ten blocks on a table, so that all of her students can see them.]

## Exhibit 1.3

After that, there is no sound . . . just quiet. . . . In the silence Tinkerbell begins to think.

These are some of the things that Tinkerbell thinks [and that Doris describes while pretending to be in a dream with her eyes closed]: The stone pieces must be used with the magic place value chart on the floor. The hint "Talking Bulldozer adds" must mean that a bulldozer is to move the stone pieces and not me. I also think that the bulldozer will have to talk and explain everything that is occurring because this is Thoughtful Mountain. The bulldozer should add the orange stone pieces together. There are three columns on the magic place value chart and three types of stone. Each type of stone piece must go in its own column—nothing should ever be in the wrong column. Hundreds must go in the hundreds column, tens must go in the tens column, and ones must go in the ones column. Ten little stone cubes are the same size as a stone long, and ten stone longs are the same size as a stone flat. I wonder if there is a special way that a talking bulldozer adds stones.

[Opening her eyes and coming out of her dream, Doris continues.] Suddenly a jingle pops into Tinkerbell's mind that she once heard children sing. It goes something like this. [Doris now sings "Tinkerbell's Addition Song," after hanging its words up on her wall on a large piece of chart paper.]

Tinkerbell thinks for a long time before deciding what to do. She does not want to make a mistake and be turned into stone, like Gandalf. And this is what Tinkerbell does.

Doris now provides her class with a demonstration, which they will later imitate. She puts the place value chart, which she previously showed her students on the table, next to the base ten blocks. She positions it so that everyone can easily see it. She then performs the following activities on the place value chart while she speaks in such a way that her students can see the correspondence between her verbalizations and the actions she performs with base ten blocks on the place value chart.

Tinkerbell gets ready to do magic: [Doris signals and magic clapping.] "Brump, flump, clump." [more magic clapping] All of a sudden, Tinkerbell turns herself into a bulldozer.

## **Tinkerbell's Addition Song**

Ones in ones

Tens in tens

Hundreds in hundreds

From right to left

And down each column we go

Down each column we go

Column by column

Column by column.

Slide them down

Then add them up

Make our trades

Each column we add

And down each column we go

Down each column we go

Column by column

Column by column.

Leave some behind

Some go up and over

All in their place

Al I I

Always legal

And down each column we go

Down each column we go

Column by column

Column by column.

Doris now pulls a plastic bulldozer from her book bag and holds it up for her class to see. She purchased it from a local toy store. Its scoop is almost the same width as the columns on the place value chart. (A third grade teacher, Laura McBride, who told "The Wizard's Tale" to her class, made bulldozers inexpensively. She cut shoe boxes in half to make two bulldozer scoops that she spray painted. Each bulldozer was about 4 by 4 by 4 inches, or 10 by 10 by 10 cm.)

## Exhibit 1.4



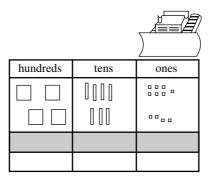
With a great roar of her engines Tinkerbell the Bulldozer drives over to one pile of stones, picks them up, and deposits them in a row on the cavern's magic place value chart. She is careful never to let a stone fall into the wrong column. [Doris acts this out with the bulldozer and the base ten blocks on the table, making loud "vrrroom" sounds whenever she moves the bulldozer forward.] Tinkerbell the Bulldozer then does the same for the other pile of orange stone pieces. [Doris also acts this out with the bulldozer and the base ten blocks with accompanying sounds.] The orange stones are then arranged on the magic place value chart like this.

Exhibit 1.5

| hundreds | tens | ones |
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Tinkerbell roars her motors, backs off so that she is above the ones column with her shovel pointing toward the column, and says some magic words so that her shovel adjusts itself in size so that it is the same width as the ones column. [The diagram below shows how Doris positions her bull-dozer (that represents Tinkerbell) with respect to the ones column.]

Exhibit 1.6



Tinkerbell, the talking bulldozer, then yells out "Ones!" and moves slowly forward pushing all of the orange stone cubes before her until they are in the shaded box near the bottom of the ones column. [Doris acts this out with the bulldozer and the base ten blocks on the till, making loud

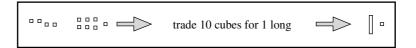
"vrrroom" sounds whenever she moves the bulldozer forward.] Tinkerbell now has eleven stone cubes in her scoop. [This is how the stones are now located above the place value chart. The arrow indicates where Tinkerbell traveled.]

Exhibit 1.7

| hundreds | tens | ones    |
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Tinkerbell calls out, "Seven cubes plus four cubes equals eleven cubes." She then yells, "Trade ten ones for one ten." To her delight, ten of the orange stone ones disappear in a puff of smoke and an orange stone long falls out of the ceiling and into the scoop of her bulldozer. [Doris demonstrates this trade.]

Exhibit 1.8



"Ah," Tinkerbell exclaims aloud, "Eleven cubes are the same as one long and one cube." [This is how the stones (base ten blocks) are now located on the place value chart.]

Exhibit 1.9

| hundreds | tens | ones |
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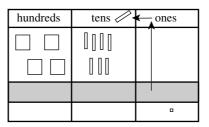
Carefully Tinkerbell drives forward a little and drops the stone cube in her scoop into the small rectangular area at the bottom of the magic place value chart, making sure not to let the long fall into the ones column. [Doris demonstrates this.] She says, "One cube." [This is how the stones are now positioned. The arrow indicates where Tinkerbell traveled.]

Exhibit 1.10

| hundreds | tens | ones       |
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Tinkerbell then backs up the ones column, and when she reaches the top where the writing is, she rotates 90 degrees clockwise and drops the stone long into the tens column on top of the word "tens"—making sure that it is entirely in the small rectangular area surrounding the word "tens." [Doris acts this out with the bulldozer and the base ten blocks on the till. This is how the till and stones now look. The arrow indicates Tinkerbell's travel route.]

Exhibit 1.11



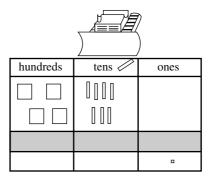
As the one stone long that is carried over to the tens column hits the magic place value chart, Tinkerbell says, "One long carried over into the tens column." [This is how the till and stones now look.]

## Exhibit 1.12

| hundreds | tens 🧷 | ones |
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Tinkerbell then backs herself around so that she faces down the tens column. [The diagram below shows how Doris positions her bulldozer.]

Exhibit 1.13



Tinkerbell now roars her engine with a great "roarrrr," yells out "Tens!" and pushes the stone longs down the tens column to the shaded rectangular area near the bottom of the column.

Exhibit 1.14

| hundreds | tens        | ones |
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|          |             |      |

Then Tinkerbell yells, "Eight longs and thus no trades!" She then moves slightly forward and dumps the eight stone longs in her scoop into the small rectangular area at the bottom of the magic place value chart and exclaims, "Eight longs!"

Exhibit 1.15

| hundreds | tens     | ones |
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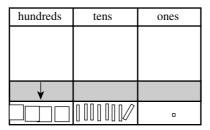
Tinkerbell then backs up the tens column and moves herself around so that she is facing down the hundreds column. She roars her engine with a great "roarrrr," yells out "Hundreds!" and pushes the stone flats down the hundreds column to the shaded rectangular area near the bottom of the column. Tinkerbell then yells, "Four flats and no trades!"

Exhibit 1.16

| hundreds | tens | ones |
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Next she moves slightly forward and dumps the four stone flats in her scoop into the small rectangular area at the bottom of the place value chart and calls out, "Four flats!" [Doris demonstrates this, making loud "vrrroom" sounds.]

#### Exhibit 1.17



Tinkerbell then spins herself around, drives off of the magic place value chart and turns herself back into her normal fairy Tinkerbell form and yells, "Four flats, eight longs, one cube!"

As soon as she says that, the color of the room flashes pink and purple for one full minute as the stones on the floor's place value chart float up into the air and disappear by quietly exploding into glorious sprays of red, green, and blue fireworks. [While saying this Doris clears the place value chart of base ten blocks.]

Tinkerbell feels very proud of herself. But when the lights stop flashing . . . two more sets of stones fall out of the ceiling, representing the numbers 467 and 355.

Doris writes 467 and 355 on the chalkboard, asks her students which base ten blocks should be set out to represent these numbers, and, under the guidance of her students, puts out the appropriate blocks in two piles next to the place value chart.

Tinkerbell now prepares to do magic. [Doris signals and magic clapping occurs.] "Brump, flump, clump." [more magic clapping] Tinkerbell again turns herself into a bulldozer, roars her engines, and moves off to solve the problem while singing the "Addition Song."

Doris now gives her students copies of "Tinkerbell's Addition Song" and has her class sing the song with her, as loud as they can, while she points to its words on the larger copy hanging on the wall.

After singing, Doris asks her students what they think the song's words mean, for the words guide Tinkerbell's actions. Doris reinforces correct interpretations of the song's words and correctly rephrases incorrect interpretations.

Next, Doris demonstrates with base ten blocks how to solve the problem 467 + 355 by telling the story of how Tinkerbell places base ten blocks on the place value chart, solves the problem column by column, calls out in words the result of each action, and announces the final answer. During this demonstration, Doris asks students the following key questions, which they answer and which she in turn clarifies:

- Can cubes go in any other columns than the ones? Can longs go in any other column than the tens? Can flats go in any other column than the hundreds?
- Do the base ten blocks that represent each of the two numbers in the problem have to be placed in rows on the upper part of the place value chart in such a way that they can be seen as two separate numbers?
- Can the bulldozer push base ten blocks down more than one column at a time? (This is critical to clarify, for many children want to push all of the blocks to the bottom of the chart at once and then trade, but this does not parallel the addition algorithm she teaches.)
- Must the bulldozer stop in the shaded rectangle, called the thinking and trading area, to
  make trades before proceeding to dump blocks at the bottom of each column? (Doris
  emphasizes that this thinking step is important.)
- When cubes or longs are traded for longs or flats, can the traded blocks be placed anywhere other than on the words at the top of their respective columns? (This is important if actions performed are to parallel written work.)

When this problem is complete, one more set of blocks falls from the ceiling representing 383 + 278. Doris tells her students that they have to solve this problem themselves, with her guidance, in order to help Tinkerbell.

Doris gives each group of two students base ten blocks, a place value chart, and a plastic bulldozer. She then carefully leads her class through the process of solving the problem, by providing verbal guidance while her students act out the problem using the bulldozers, base ten blocks, and place value charts. Doris's method of proceeding parallels the demonstration of the addition process that she has previously provided her students. As they work on the problem, Doris

- first asks students to sing "Tinkerbell's Addition Song," and then later during the addition process she asks them to sing relevant parts of the song,
- asks students to describe what to do at each step of the problem,
- clarifies responses to her questions received from students,
- encourages students to make sounds, such as "vrrroom, vrrroom" while moving their bulldozers forward down columns, and
- monitors students' actions to correct any incorrect behavior.

Doris encourages students to work together cooperatively because doing so will help Tinkerbell. She tells them that they must work cooperatively with their partner and help their partner whenever possible, for the success of their group in helping Tinkerbell depends on both partners being able to solve the mathematics problem and not just one of them. She tells them that part of helping each other involves quietly saying out loud all of their mathematical thoughts. This means verbalizing or describing in words their actions and the reasons for their actions. Doris tells her students that they must also carefully listen to what each other says and check each other's manipulations with the blocks. Doris emphasizes that if they think their partner makes a mistake or if they do not understand why their partner does or says something, it is their job to politely and respectfully ask their partner questions and make sure that they both understand, either by teaching each other or getting help from her.

When the problem is complete, Doris continues the story:

When the third problem is complete, the room flashes from pink to purple to red over and over again for one full minute as the stones on the cavern's magic place value chart disappear. Then the room becomes completely dark and the mountain speaks to Tinkerbell, "Very good, little fairy. You can now either be released from me to go in peace, or have any wish that is in my power to grant."

Tinkerbell yells, "My wish is that you release my friend Gandalf!"

There is a great sound of breaking stone and Gandalf falls out of the stone wall, fully human again. Tinkerbell examines Gandalf closely to see if his true essence was changed in any way. Is he still wise, and will he still rather share his wisdom with some boys on a baseball field than be in a television program? Is he still gentle, and will he still refuse to step on ants? Is he still honest, and will he do magic to find the owner of a lost dollar rather than keep it himself? Is he still the champion of all creatures in need, who would rather save a mouse in need of help than be cheered at a king's banquet?

When Tinkerbell sees that the person who fell out of the wall is the same Gandalf she has loved and respected for years, she flies to him, pulls on his beard, and yells, "Gandalf, Gandalf, how did you ever get yourself into this predicament?"

Gandalf tells her how smart she is to have figured out how to do the mountain's problems and that he came to save one of his good friends, Habble. Habble is one of the stone wizards who is frozen in the cavern's wall.

Before they can say more, the mountain rumbles and booms, and Tinkerbell and Gandalf prepare themselves for something earth-shattering.

Doris now stops telling the story and tells her students that she will continue it tomorrow. She then asks them to discuss in their small groups two of the following questions and to record their answers on a sheet of paper:

- How did the story make you feel (and why)?
- What did the story remind you of in your own life?
- Did you like the story (and why)?
- If you were to tell a friend about the story, what are two things that would be most important for your friend to know (and why)?
- Do you think you know what will happen at the end of the story (and if so, what is it)?

Before discussion begins, Doris emphasizes the importance of cooperatively discussing the questions and listening to each other, learning from each other, and teaching each other. Doris stresses that the success of their group depends on both members of the group contributing to the discussion rather than just one person dominating the discussion.

During the discussions Doris circulates among students reinforcing cooperative behavior and stimulating discussion. Afterward, she draws students together for a whole-class discussion during which selected small groups report their results by reading and explaining what they recorded.

## DAY 2

This is the third year that Doris is presenting "The Wizard's Tale," and she tells the story from memory, only occasionally referring to notes. She says that this telling of the story is different from the last two. She feels freer to alter, elaborate upon, and embellish the story—to stimulate specific interactions with her class or elaborate on events that occur in her classroom. (One of the ancient traditions of oral storytelling is the altering of stories to stimulate audience interaction and involvement.)

Multidigit addition is part of the second and third grade curriculum in Doris's school. Doris has found, however, that her fourth graders arrive knowing the procedures required for multidigit addition but that they do not have a good understanding of why they juggle digits as they do during addition—that is, they have procedural skills but inadequate conceptual understanding. She wants her students to acquire conceptual understanding, efficient performance of procedural skills, and the ability to relate the two to each other. Therefore, she reteaches addition.

When Doris announces that she is about to continue "The Wizard's Tale," her students give a yelp of delight. Doris tells them to get a partner and a tub of base ten blocks, a bull-dozer, and a place value chart from her storage area. When students complete this and are sitting quietly, she begins.

"The Wizard's Tale" ended yesterday in Thoughtful Mountain with Tinkerbell sitting on Gandalf's beard. Before they can say much to each other, the mountain begins to rumble and boom.

In fear, Gandalf throws himself on the floor, flattens himself against it, puts his hands over his head, and puts a protective magic spell on himself so that nothing can hurt him. Tinkerbell, who is caught between his scratchy beard and the cave's hard floor, starts yelling to Gandalf, "Get up, it is the mountain speaking to us!" Slowly Gandalf gets up and Tinkerbell frees herself from his scratchy beard.

During the rumbling and booming, the cave's walls start to glow red, and the mountain says in a deep voice, "No-talking bulldozer, talking giant parrot!" Then the cave's ceiling starts to crackle. From two different parts of the ceiling, orange pieces of stone fall to the floor. This is what the two sets of stones look like.

## Exhibit 1.18



[Doris puts these base ten blocks on a classroom table where everyone can see them. She asks her students how to say what is in each set of blocks.]

Next the cave's floor starts to glow pink, and this magic place value chart appears on it, etched in the stone. [Doris holds up a place value chart and then places it on the table with the base ten blocks.]

Exhibit 1.19

| hundreds | tens | ones |
|----------|------|------|
|          |      |      |
|          |      |      |
|          |      |      |
|          |      |      |
|          |      |      |

Then Thoughtful Mountain's rumbling and booming cease, and the lights go out. Tinkerbell and Gandalf are left in the dark. Tinkerbell fires up her magic halo of stars so that it glows brightly. Gandalf gets ready to say some magic words. [Doris signals and magic clapping occurs.] "Lit, flitt, latt, flight." [more magic clapping] Suddenly a small moon begins to glow above Gandalf's head. There, in the dim starlight and moonlight, the two wizards stand, wondering what to do.

Gandalf says, "I think we need to figure out what the mountain's words mean."

Tinkerbell agrees and repeats the words, "No-talking bulldozer, talking giant parrot!" She says, "It seems that we will need both a bulldozer that cannot talk and a talking parrot. Since I have already been a bulldozer, why don't I be the bulldozer and you be the talking parrot."

"No way," says Gandalf, "I don't want to be a bird. Parrots have fleas. No way do I want to be a feather-covered, flea-infested parrot."

"Would you rather be a rusty, old metal bulldozer?" asks Tinkerbell.

"I don't want to be that either," exclaims Gandalf.

"So then be the talking parrot," says Tinkerbell. "I'll turn you into the most beautiful parrot there ever was. I'll give you a beautiful coat of blue, red,

and gold feathers that shine brilliantly. I'll make sure you have no fleas. All you will have to do is talk like a parrot in a squeaky voice!"

Gandalf agrees to be the parrot.

The two wizards now plan how they will use the pieces of stone on the cave's floor to solve the mountain's addition problem. They plan how Tinkerbell will act as a bulldozer who cannot say anything as it moves the stones. They plan how Gandalf will act as a giant parrot and verbalize the result of each of Tinkerbell's actions. As they plan, Tinkerbell sings Gandalf her "Addition Song." Let us sing it for him also.

[Doris has students get out their copies of "Tinkerbell's Addition Song" and sing (or chant) it.]

Tinkerbell and Gandalf now discuss the meaning of the song's words and how Gandalf might describe in words each of Tinkerbell's actions. Then Tinkerbell and Gandalf go into action.

Tinkerbell says the magic words [Doris signals and magic clapping occurs.] "Crack, cronk, crooky." [more magic clapping] Suddenly Gandalf turns into the most beautiful parrot there ever was, with a beautiful coat of blue, red, and gold feathers that shines brilliantly, that has no fleas, and that speaks in a squeaky voice.

[Doris pulls a multicolored wood parrot perched on a wood dowel out from her book bag and holds it up for her class to see. She found two dozen of these parrots in her custodian's storage room. They had once been used during a fundraiser.]

Tinkerbell then says some more magic words [Doris signals and magic clapping], "Brump, flump, clump." [more magic clapping] Suddenly Tinkerbell turns into a bright yellow bulldozer. [Doris holds up her yellow plastic bulldozer for her class to see.]

During the following demonstration, whenever Doris acts out Tinkerbell's actions with base ten blocks, she has a hand on her yellow bulldozer. Whenever she utters Gandalf's parrot verbalizations, she holds up the wood parrot and speaks in a squeaky voice. She performs the demonstration on the table in front of the classroom in such a way that her students can see the correspondence between her verbalizations and actions.

With a roar Tinkerbell drives over to one pile of orange stones, picks them up, and deposits them on the floor's magic place value chart, being careful to never let a stone fall into the wrong column. If she does, she might be turned into stone by the mountain. [As she speaks, Doris begins doing this.]

While Tinkerbell is moving the orange stones, Gandalf the Giant Talking Parrot says in a squeaky voice, "We take our first group of orange stones and place them on the magic place value chart so that the three flats are in the hundreds column, the four longs are in the tens column, and the five cubes are in the ones column. This makes three hundred forty-five." [While saying this Doris speaks in a squeaky voice while holding up the parrot.]

Tinkerbell then does the same for the other pile of orange stone pieces [using the bulldozer] while Gandalf the Giant Talking Parrot says in a

squeaky voice, "We take our next group of stones and place them on the magic place value chart so that the two flats are in the hundreds column, the six longs are in the tens column, and the eight cubes are in the ones column. This makes two hundred sixty-eight. [Again Doris holds up the parrot while speaking in a squeaky voice. The stones are arranged like this.]

Exhibit 1.20

| hundreds | tens  | ones |
|----------|-------|------|
|          | 0000  |      |
|          | 00000 |      |
|          |       |      |
|          |       |      |

Gandalf then announces, "Now we add three hundred forty-five and two hundred sixty-eight. First we add all of the cubes in the ones column." [Again, Doris holds up the parrot when speaking in a squeaky voice.]

Tinkerbell now roars her motors [Doris makes "vrrroom" sounds] and backs up so that she is above the ones column with her shovel pointing toward the column. Tinkerbell the Bulldozer now moves slowly forward pushing all of the orange stone cubes before her until they are in the shaded box near the bottom of the ones column. This box is called the thinking and trading area. Tinkerbell now has twelve stone cubes in her scoop. Gandalf the Talking Parrot calls out in a squeaky voice, "Five cubes plus seven cubes equals twelve cubes." He then squawks, "Trade ten ones for one ten." Ten of the ones disappear in a puff of smoke and a long falls out of the ceiling and into Tinkerbell's bulldozer's scoop. [Doris acts this out in front of the class, trading 10 cubes for a long.] Gandalf the Giant Talking Parrot squeaks, "Twelve cubes is the same as one long and two cubes." Carefully Tinkerbell drives forward a little [Doris makes "vrrroom" sounds] and drops the two stone cubes in her scoop into the small rectangular area at the bottom of the magic place value chart, making sure not to let the long fall into the ones column. Gandalf squeaks, "Two cubes are left in the ones column and one long gets carried to the top of the tens column." As he says this Tinkerbell backs up the ones column, and when she reaches the top where the writing is, she rotates clockwise 90 degrees and drops the stone long into the tens column on top of the word "tens"—making sure that it is entirely in the small rectangular area surrounding the word. [Doris acts this out with the bulldozer while making "vrrroom" sounds.] As the one stone long that was carried over to the tens column hits the magic place value chart, Gandalf squeaks, "One long has been carried over into the tens column."

Doris now repeats this demonstration for the stones in the tens and hundreds column of the place value chart. When she finishes, she continues the story.

As Tinkerbell backs off of the magic place value chart, Gandalf the Giant Talking Parrot squeaks, "Three hundred forty-five plus two hundred sixty-eight equals six hundred thirteen."

As soon as Gandalf says that, the color of the room flashes pink and purple for one full minute as the stones on the cavern's magic place value chart float up into the air and disappear by quietly exploding into glorious sprays of red, green, and blue fireworks. [While saying this, Doris clears the place value chart of base ten blocks.]

Tinkerbell and Gandalf feel very proud of themselves. But when the lights stop flashing . . . two more sets of stones fall out of the ceiling, representing the numbers 446 and 378.

Doris asks her class which base ten blocks should be set out to represent these numbers. She calls on students to answer, and under their guidance lays out the blocks on the table next to the place value chart.

Doris tells her students that in their groups of two they must help Tinkerbell and Gandalf by acting out the second problem given by Thoughtful Mountain as though they are the wizards. One member of each group must pretend to be Tinkerbell, use the bulldozer, and move the base ten blocks. The other member of the group must pretend to be Gandalf the Giant Talking Parrot, hold up the wood parrot when speaking, and be the *verbalizer*, who describes in words all of the actions taken by the bulldozer. Doris tells her students that there will be a third problem given by the mountain and that they will switch roles for the third problem so that each person in their group will have a chance to be Tinkerbell and Gandalf.

Doris has her students choose roles as she distributes wood parrots. Next, she has her students sing "Tinkerbell's Addition Song" and reminds them of the meaning of some of its phrases. She also reminds them that they are to work cooperatively, what this means, and that if someone needs help to gently and kindly teach them what they need to know in the same way that Tinkerbell would. Doris's students then start working the problem 446 + 378. Base ten blocks are moved about by plastic bulldozers as children make "vrrroom" and "beep, beep, beep" sounds (of bulldozers going forward and backward). Parrots are raised into the air as squeaky voices verbalize the mathematics being performed by the bulldozer. Students monitor each other's behavior and discuss what to do next as they refer to "Tinkerbell's Addition Song."

Doris circulates among her students while they work, observing them, correcting their base ten blocks manipulations and their verbalizations, and reminding them (as needed) what it means to work cooperatively (by treating each other with respect as Tinkerbell would, and remembering that what is important is the success of both members of the group and not just the ability of one member to get a correct answer).

When her students complete the second problem, Doris asks them to switch roles as Tinkerbell and Gandalf and gives each group the problem 275 + 188.

As groups complete the third problem, Doris asks students to discuss in their small groups how the work they did relates to mathematics they might have learned elsewhere. (Doris's students had previously constructed "invented algorithms" for addition.) Doris reminds her

students that they are helping Tinkerbell in Thoughtful Mountain, and they must be thoughtful and must share their thoughts with the other members of their group so that all members of their group can learn from their reflections. Doris's students record their thoughts as they discuss.

When small-group discussions are complete, Doris holds a whole-class discussion. During the discussion Doris relates her students' comments to "Tinkerbell's Addition Song" and her addition demonstration. Some of the issues commented on include place value, addition, regrouping, trading between columns, working column by column, and working from right to left versus left to right. A debate arises over whether it is best to do addition from right to left or left to right. The following issues are raised: Does it make a difference? Which direction requires the least backtracking of work? Which direction is most efficient? And what is our cultural convention? After the discussion, Doris continues the story.

When Tinkerbell and Gandalf complete the third problem with your help, the cavern in Thoughtful Mountain flashes pink and purple for two full minutes as the stones on the cavern's place value chart float up into the air and disappear by quietly exploding into glorious sprays of red, green, and blue fireworks. [While saying this Doris clears the place value chart of base ten blocks.]

Then the cavern becomes completely dark and the mountain speaks to Tinkerbell and Gandalf in a deep voice, "Very good, little Wizards. You can now either be released from me to go in peace, or have any wish that is in my power to grant."

Gandalf exclaims, "Release my friend Habble!"

There is a great sound of breaking stone as Habble falls out of the stone wall, fully human again. Gandalf walks over to him, gives him the secret wizard's foot-shake, and asks him why he came into Thoughtful Mountain. [Doris demonstrates a special foot-shake to her students that includes tapping ankles together.]

Tinkerbell examines Habble's essence. She discovers that he can be trusted, that he is a lover of flowers, that he knows better than any other wizard how to speak their true speech, and that he will always be loyal to his friends. She also discovers that Habble is absentminded. He can forget to put on his shoes for days at a time or can start out to visit a friend for dinner, get distracted by a flower, sit down and talk with it, and forget that he is going to dinner. Habble tells Gandalf and Tinkerbell that he came to Thoughtful Mountain to save one of his friends, Bondo. Bondo is one of the stone wizards in the cavern's wall.

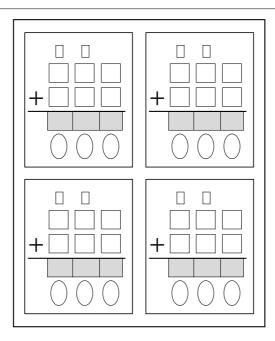
Before they can say much, the mountain begins rumbling and booming, and Tinkerbell, Gandalf, and Habble get prepared for something earth-shattering.

## DAY 3

During this episode of "The Wizard's Tale," Doris puts her students in groups of three, with base ten blocks, plastic bulldozers, wood parrots, place value charts, chartreuse crayons, and

worksheets with four magic addition graphics on each. (See the adjacent sample sheet.) When they are in groups with the necessary materials, Doris continues the story.

Exhibit 1.21



"The Wizard's Tale" ended yesterday with Tinkerbell, Gandalf, and Habble talking in Thoughtful Mountain. Before they could say very much, the mountain began to rumble and boom.

Habble falls to his knees in terror, his hair stands on end, and he starts yelling, "Mama! Mama!" Tinkerbell shouts, "Stop yelling for your mother and get up. It is the mountain speaking to us!" Habble quiets down and gets up. As the mountain continues to boom and rumble, Tinkerbell, Gandalf, and Habble stand quietly.

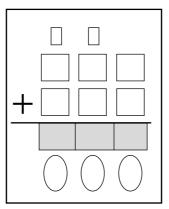
Suddenly, the booming and rumbling stop as the cave's walls start to glow red and the mountain [*Doris*] says in a deep voice, "No-talking bulldozer, mathematically talking giant parrot, writing gorilla!"

Then the cave's ceiling starts to crackle. From three different parts of the ceiling orange pieces of stone fall to the floor. There is a pile of cubes, a pile of longs, and a pile of flats. [Doris places a pile of each of these types of base ten blocks on her demonstration table.]

Next the floor of the cave starts to glow pink, and a magic place value chart appears on it, etched in the stone. [Doris holds up a place value chart, then puts it on the table with the base ten blocks.]

Then one of the cave's walls starts to glow violet, and loud crunching sounds can be heard as hundreds of ants appear to be eating away at it. Out of the wall gradually appears a violet-colored, magic addition graphic. Next to the magic addition graphic there also appears a small stone shelf with a chartreuse "stone pencil" on it, with these words inscribed on it, "Use me to write on the wall." This is what the magic addition graphic looks like. [Doris hangs a large copy of the addition graphic, drawn with a violet crayon, on the wall behind the demonstration table. It is the same as the four on the student worksheet.]

Exhibit 1.22



Then the lights go out, and Tinkerbell, Gandalf, and Habble are left in the dark. Tinkerbell fires up her magic halo of stars so that it glows brilliantly while Gandalf makes the small moon above his head glow brightly.

There, in the starlight and moonlight, the three wizards wait until Gandalf says, "I think we need to figure out what the mountain's words mean."

Tinkerbell agrees and repeats the words: "No-talking bulldozer, mathematically talking giant parrot, writing gorilla!" Then she says, "I think we need a bulldozer that cannot talk, a mathematically talking parrot, and a writing gorilla. Since I have already been a bulldozer and Gandalf has already been a giant talking parrot, why don't we make Habble a writing gorilla!"

The three wizards now plan how they will work together. Tinkerbell the Bulldozer will move the stones about without saying anything. Gandalf the Mathematically Talking Parrot will describe in words each action that Tinkerbell takes. And Habble the Writing Gorilla will use the chartreuse stone pencil to record on the addition graphic what Tinkerbell does and Gandalf says. Tinkerbell sings Habble her "Addition Song," they discuss the meanings of its words, and they discuss how Gandalf might verbally describe in words each of Tinkerbell's actions and how Habble might write the result on the addition graphic with the chartreuse pencil.

Doris tells her students that the wizards will benefit from their thoughts, so they must sing "Tinkerbell's Addition Song" and then discuss the same things the wizards are discussing. Afterward, they will telepathically send their thoughts to the wizards. During the discussion Doris highlights issues of how a mathematically talking parrot is different from a talking parrot; whether in mathematical language they should say "cubes" or "ones," "longs" or "tens," and "flats" or "hundreds;" how the magic addition graphic is similar to the magic place value chart (both with their columns and thinking and trading areas), and what it is that should be written on the addition graphic. Doris ends the discussion by having her class telepathically send their thoughts to the wizards by participating in the ritual of clapping three times, concentrating on the important issues of the discussion, and then clapping three more times.

While discussing how to add, the wizards all have the same thoughts right after we telepathically send them our thoughts. As a result they decide that Gandalf the Talking Parrot should talk proper mathematical language and use the words "ones" instead of "stone cubes," "tens" instead of "stone longs," and "hundreds" instead of "stone flats." They decide that Habble should only write numbers with the chartreuse stone pencil, and that the numbers should only go in the boxes, ellipses, and thinking and trading area of the addition graphic. They also decide that each base ten block manipulation should be immediately followed by the talk and writing that go with it.

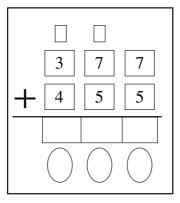
When the wizards finish their discussion, they go into action.

Gandalf gets ready to do magic. [Doris signals and magic clapping occurs.] "Crack, cronk, crooky," [more clapping] and Gandalf turns himself into a beautiful parrot that speaks in a squeaky voice. Tinkerbell gets ready to do magic. [Clapping] "Brump, flump, clump," [more clapping] and Tinkerbell turns herself into a bulldozer. Habble does not know how to do form changing magic, so Tinkerbell does magic on him. [Clapping] "Habble, gore, gorie, gorum," [more clapping] and Habble becomes a hairy gorilla with long arms and a bright pink nose.

During the following demonstration, whenever Doris acts out Tinkerbell's actions with base ten blocks, she does so with her yellow plastic bulldozer. Whenever she utters Gandalf's squeaky verbalizations, she holds up the wood parrot. Whenever she writes numbers for Habble, she uses the chartreuse marker on the addition graphic hanging on the wall. She performs the demonstration on the table in front of the classroom in such a way that her students clearly see the correspondence between her actions, verbalizations, and writing.

As soon as the wizards change their forms, a screeching sound comes from the addition graphic on the wall and the numbers 377 and 455 appear in it, like this. [Doris writes the numbers with her chartreuse crayon while saying this.]

## Exhibit 1.23

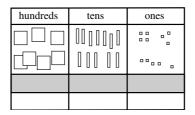


Gandalf immediately squawks that 377 is three hundred seventy-seven, which is three hundreds, seven tens, and seven ones.

With a roar of her engine ["Vrrroom," says Doris] Tinkerbell drives over to the pile of hundreds and picks up three, then to the tens and picks up seven, and then to the ones and picks up seven. As she does this Gandalf [Doris] squeaks "three hundreds" when she picks up the hundreds, "seven tens" as she picks up the tens, and "seven ones" as she picks up the ones. Habble points at each of the digits in the addition graphic as they are uttered [as does Doris]. Tinkerbell the Bulldozer [accompanied by Doris's "vrooms"] now deposits the stones in the cave's magic place value chart being careful to never let a stone fall into the wrong column. As she does this, Gandalf the Mathematically Talking Parrot squeaks, "We take our first group of stones and place them on the place value chart so that the three hundreds are in the hundreds column, the seven tens are in the tens column, and the seven ones are in the ones column. This makes three hundred seventy-seven." As this is done Habble [Doris] points to the corresponding digits in the addition graphic.

The wizards (and Doris) then do the same for the number 455 and base ten blocks are arranged on the demonstration table's place value chart like this.

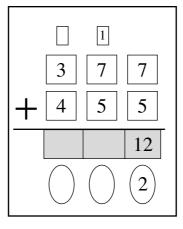
Exhibit 1.24



Gandalf then squeaks, "Now we add three hundred seventy-seven and four hundred fifty-five. First we add the ones."

Tinkerbell roars her motor, moves so her shovel points down the ones column, and moves forward collecting all of the ones until her scoop is above the shaded thinking and trading box near the bottom of the column. Tinkerbell now has twelve ones in her scoop. Gandalf squeaks, "Seven ones plus five ones equals twelve ones." As Gandalf says this, Habble writes "12" in the thinking and trading area in the ones column. Gandalf then squawks, "Trade ten ones for one ten." Ten of the ones disappear in a puff of smoke and a ten falls out of the ceiling and into Tinkerbell's bulldozer scoop. Gandalf the Mathematically Talking Parrot squeaks, "Twelve ones is the same as one ten and two ones." Carefully Tinkerbell drives forward a little and drops the two ones near the bottom of the ones column. Gandalf squeaks, "Two ones are left in the ones column, and one ten gets carried to the tens column." As Gandalf says this, Habble writes a "2" in the ellipse in the ones column to represent the two ones. Next Tinkerbell backs up the ones column [as Doris's students make "beep, beep, beep" sounds], and when she reaches the top where the writing is, she rotates clockwise 90 degrees and drops the stone ten into the tens column on top of the word "tens." As it hits the magic place value chart, Gandalf squeaks, "One ten has been carried over to the tens column," and Habble writes a "1" in the small box at the top of the tens column in the addition graphic to represent that ten. This is what the addition graphic now looks like:

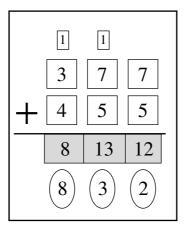
Exhibit 1.25



Doris now repeats this demonstration for the tens and hundreds columns. When she finishes she continues.

As Tinkerbell backs off of the magic place value chart on the floor, Gandalf squeaks, "Three hundred seventy-seven plus four hundred fifty-five equals eight hundred thirty-two," as Habble points to this in the wall's addition graphic.

Exhibit 1.26



As soon as Gandalf says that, the color of the room flashes pink and purple as the stones on the place value chart float up into the air and disappear by exploding into glorious sprays of red, green, and blue fireworks and as the numbers in the wall's addition graphic disappear. [While saying this, Doris clears the place value chart of base ten blocks and puts up a new addition graphic.]

Tinkerbell, Gandalf, and Habble feel very proud of themselves. But when the lights stop flashing . . . the numbers 626 and 295 magically appear in the addition graphic as firecrackers explode next to it.

Doris asks her students which base ten blocks should be set out to represent these numbers. Then she asks them to show her in each of their groups on their worksheet with a chartreuse crayon what should be written in the rectangles of one of the addition graphics.

Doris tells her students that in their groups of three they must help the wizards by acting out the second problem given by Thoughtful Mountain as though they are the wizards. One member of each group must pretend to be Tinkerbell, use the bulldozer, and move the base ten blocks. Another must pretend to be Gandalf the Giant Talking Parrot, hold up the wood parrot when speaking, and be the verbalizer, who describes in words Tinkerbell's actions. The other group member must pretend to be Habble the Writing Gorilla and use a chartreuse crayon to record numbers in one of the addition graphics on the worksheet, numbers that describe Tinkerbell's actions and Gandalf's verbalizations. Doris tells her students that there will be a third and fourth problem given by the mountain and that they will switch roles for these problems so that each group member will have a chance to be Tinkerbell, Gandalf, and Habble.

Doris then has her students choose roles and get the symbol of their role: a bulldozer for Tinkerbell, the wood parrot for Gandalf, and the chartreuse crayon for Habble. Next, her students sing "Tinkerbell's Addition Song." Doris reminds them of the meaning of some of its phrases and that they are to work cooperatively and what this means.

Doris's students go right to work on the problem 626 + 295. Base ten blocks are moved about by plastic bulldozers as children make "vrrroom" and "beep" sounds, parrots are raised into the air as squeaky voices verbalize mathematics, and chartreuse crayons are carefully applied to worksheets. Students monitor each other's behavior and discuss what to do next. As before, Doris circulates among her students observing them, providing help as needed, asking questions, and reminding them what it means to work cooperatively.

As students complete the second problem, Doris asks them to switch roles as Tinkerbell, Gandalf, and Habble and gives each group the problem 255 + 366. When the third problem is solved, students again switch roles and work the problem 394 + 177.

As before, when groups complete the third problem, Doris asks students to discuss in their small groups how the work they did relates to mathematics they might have learned elsewhere and to record their conclusions. When small group discussions are complete, Doris holds a whole-class discussion during which students share their thoughts.

During the discussion Doris focuses attention on correct mathematical language. For example, she points out that for a problem like 372 + 251, when adding the 7 and 5 in the tens column, one *should not* say, "7 plus 5 equals 12, leave 2 and carry 1," but that one *should* say, "7 tens plus 5 tens equals 12 tens. Regroup 10 tens for 1 hundred so that we have 1 hundred and 2 tens. Leave the 2 tens in the tens column and carry the 1 hundred over to the hundreds column." Doris emphasizes that simply saying, "7 plus 5 is 12, leave a 2 and carry 1" is inadequate, for it does not highlight the meaning of the mathematics—which is required by Thoughtful Mountain in order to avoid being turned to stone. (Helping children relearn some of their mathematical language in a thoughtful way so that the language highlights mathematical meanings is an important part of this story.)

When the discussion is complete, Doris continues with the tale. As before, base ten blocks disappear, Thoughtful Mountain offers freedom or the granting of a wish, and Gandalf chooses that Thoughtful Mountain release Bondo. Then . . .

There is a great sound of breaking stone and Bondo falls out of the cavern's wall, fully human again. Habble runs over to her, gives her the secret wizard's foot-shake and a big hug, and asks her why she came into Thoughtful Mountain. As this takes place Tinkerbell studies Bondo's essence. She discovers that Bondo loves to have pretty things and that she loves to be the center of attention. That is probably why she has twelve rings on her fingers and eight different hair pins in her pony tail (one of which is a bee hive with real bees flying about it). She also loves puppy dogs, kittens, and mice. Bondo can be trusted, although she might act spoiled and silly at times.

While Tinkerbell is studying Bondo, Bondo tells the wizards that she came to Thoughtful Mountain to save her puppy dog Zunk, who had been turned into a witch and sent to Thoughtful Mountain by an evil wizard for bothering him while he was contemplating. Zunk is the stone witch who remains frozen in the cavern's wall.

Before they can say much, the mountain begins rumbling, and Tinkerbell, Gandalf, Habble, and Bondo get prepared for something earth-shattering.

Doris now ends this episode of the story.

## DAY 4

Day 4 activity is similar to day three, but with three differences. First, base ten "stones" and the place value chart are no longer used. Second, mathematical problems begin to be related to real world situations. Third, the wizards—and Doris's students—work in groups of two as a "mathematical talker" and a "mathematical writer."

Doris begins by putting her students in groups of two and giving each group a wood parrot, a chartreuse crayon, and a copy of the worksheet containing four addition graphics.

Doris starts the story by telling her students how Tinkerbell, Gandalf, Habble, and Bondo were talking when the mountain began to rumble and boom. At the sound of the rumbling, Bondo turns herself into a lion and tries to eat Gandalf, who saves himself with some magic words that turn Bondo back into herself.

Then the cavern's walls start to glow red, and in a booming voice the mountain says, "Two by two wizards go as mathematically talking giant parrot and mathematically writing gorilla!"

Next, one of the cavern's walls starts to glow white like polished marble. Across the top half of the wall a miniature marching band appears playing musical instruments. Band members are dressed in uniforms made out of green emeralds and blue sapphires and carry musical instruments made out of gold. First there is a group of musicians in a square array lined up in ten neat rows with ten musicians in each column." That's one hundred musicians!" yells Habble. Following them is a second square of one hundred musicians. Following them is a third square of one hundred musicians. Next, there are six neat rows of musicians with ten trumpet players in each row. Finally, there are eight separate musicians, each playing a big gold drum. Then the music and the musicians stop.

The wizards decide that there are three hundred sixty-eight miniature musicians in the three square arrays, six rows, and eight individuals.

After about two minutes, music starts up again and across the lower half of the wall marches a second miniature marching band whose members are dressed in white opals and purple amethysts and whose musical instruments are made of silver. As before the musicians march in square arrays, rows of ten, and as individuals. First two squares, each of which has one hundred musicians, march across the wall. They are followed by three rows of musicians, each of which contains ten trombone players. Next, five musicians enter, each playing a silver xylophone.

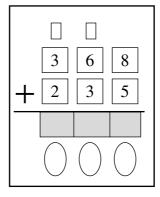
When the music and the musicians stop, the wizards discuss how many musicians there are on the lower half of the wall.

Doris asks her students to raise their right hands if they know how many musicians are on the lower half of the wall. After a pause Doris says, "On the count of three, yell out the answer all together in unison... one, two, three." Her students bellow, "Two hundred thirty-five." Doris responds, "You know what? The wizards also decide that there are two hundred thirty-five musicians."

"But why are they there?" asks Gandalf.

One of the other walls of the cavern now starts to glow violet. It appears to have hundreds of small green jade dragons eating away at it. Out of the wall gradually appears an addition graphic that glows violet. Next to the addition graphic appears a small stone shelf with a chartreuse "stone pencil" on it. Then, with a screeching sound, like chalk being drawn across a blackboard, numbers appear in the addition graphic, like this. [Doris writes chartreuse numbers in an addition graphic on a large sheet of paper hanging on her wall.]

Exhibit 1.27



Next the lights go out and the rumbling stops, and Tinkerbell, Gandalf, Habble, and Bondo are left in the dark. Tinkerbell makes her magic halo of stars glow brilliantly while Gandalf makes the small moon above his head glow brightly. There, in the starlight and moonlight, the wizards stand quietly.

Suddenly Habble yells, "I know what to do with the marching bands! The number of musicians in each band is the same as the numbers in the wall's addition problem! The number problem on the wall represents the picture problem of the musicians. We need to add the number of musicians in the two bands! Aren't I smart!"

"That makes sense," says Tinkerbell, "and I don't have to be a bulldozer again!"

Gandalf laughs as he comments, "Well, now you can be a talking parrot or a writing gorilla! The mountain said, 'Two by two wizards go as mathematically talking giant parrot and writing gorilla!' We now have to solve problems in groups of two, with a talking parrot and a writing gorilla in each group."

Tinkerbell agrees and adds, "Gandalf, why don't you and I go first so we can show Bondo what to do. Since you have done such a good job as a mathematically talking parrot, why don't you be the parrot and I'll be the writing gorilla?"

Gandalf agrees.

Bondo says loudly, "I don't know what I am supposed to do!"

The three wizards who have already done problems now sing Bondo "Tinkerbell's Addition Song" and explain how it relates to addition.

As before, Doris tells her students that the wizards will benefit from their thoughts, so they must sing "Tinkerbell's Addition Song" and then discuss the same things the wizards are discussing. During the discussion Doris highlights issues of how a mathematically talking parrot is different from a talking parrot and what should be written in the addition graphic. Doris ends the discussion by having her class telepathically send their thoughts to the wizards by participating in the clapping, thinking, clapping ritual.

While discussing how to add, the wizards all have the same thoughts right after we telepathically send them our thoughts. As a result they tell Bondo how to add numbers in each column, column by column, from right to left. First the ones, then the tens, and then the hundreds. They describe how the columns always have to be legal with only ones in the ones column, tens in the tens column, and hundreds in the hundreds column. They describe how the thinking and trading area works, how to make ten for one trades, and how to carry the traded numbers to the next column. And they make very sure that Bondo understands that when adding two numbers in the tens column, such as in the problem 34 + 52, that she must never say, "3 plus 5," but that she must say, "3 tens plus 5 tens," because the 3 and 5 really represent 3 tens and 5 tens and not just 3 and 5. They carefully explain how the place where a digit is located in a problem tells what its value is and how a mathematically correctly speaking person has to say its value when adding.

The three wizards comment that teaching Bondo how to add helps them better understand addition.

Gandalf starts to hiss as they finish teaching Bondo addition. "The mountain is playing a trick on us!" he says as he spits out twenty-seven firecrackers that explode in mid-air.

"What do you mean?" Tinkerbell asks.

"Look at the problem: 368 + 235! Look at the tens column. If we add the 6 tens to the 3 tens we get 9 tens. But we are going to have 1 ten to carry over from the ones column. So if we add this to the 9 tens, that will give us 10 tens. If we now trade 10 tens for 1 hundred, we have no tens left. That's the trick. What do we do when we have no tens?"

"We can just write nothing," says Habble.

"No way!" says Gandalf. "We have to put something in the ellipse at the bottom of the tens column. And since we don't have any tens, we have nothing to put in it."

"Why can't we just write down nothing?" asks Tinkerbell.

"How can you write nothing and have something written down?" asks Gandalf.

"Come on, silly," says Tinkerbell, "just write down 0 and then you will have written down that there is nothing in the tens column."

"But 0 is nothing, and we have to write something!" Gandalf insists.

Doris asks her students how to help Gandalf understand what to do. After taking several answers, she continues the story.

"Gandalf, come on! There is a difference between the numeral 0 that tells you that you do not have anything and the amount that you have when you have 0 amount, which is nothing," says Habble. "If we put a 0 in the tens column, it means that we have no tens. 0 is the numeral that allows you to say that you have none. If we have the number 'two hundred three' and we write it without a 0, we would just have 2, 3 which is twenty three; if we put in the 0 to say that there are no tens, then we have 2, 0, 3, which is two hundred three."

"OK, now I've got it!" says Gandalf. "Let's just make sure that whenever we have nothing as a result of adding and making our trades, that we write down the 0 to say we have none of something."

Tinkerbell says, "Let's start solving the problem, already! Gandalf and I will start out doing the first problem, and then Bondo and Habble can do the next one." Gandalf gets ready to do magic. [Doris signals and magic clapping occurs.] "Crack, cronk, crooky," [more magic clapping] and Gandalf turns himself into a beautiful parrot that speaks in a squeaky voice. Tinkerbell gets ready to do magic. [Clapping] "Brump, flump, clump," [more clapping] and Tinkerbell turns herself into a hairy gorilla with long arms and a pink nose, who writes the results of Gandalf's verbalizations with a chartreuse stone pencil.

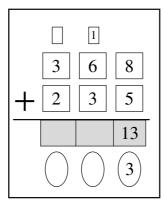
During the following demonstration, whenever Doris utters Gandalf's parrot's squeaky verbalizations, she holds up the wood parrot. Whenever she writes numbers for Tinkerbell, she uses the chartreuse marker on the addition graphic hanging on the wall. She does this in a way that highlights the correspondence between verbalizations and writing.

Gandalf squawks, "We are adding 368 and 235, which is three hundred sixty-eight plus two hundred thirty-five. 368 is 3 hundreds, 6 tens, and 8 ones. 235 is 2 hundreds, 3 tens, and 5 ones." As Gandalf the Mathematically Talking Parrot says this, Tinkerbell the Writing Gorilla points at each of the digits in the magic addition graphic.

Gandalf the Mathematically Talking Parrot now announces that they will first add numbers in the ones column. Tinkerbell points to them. Gandalf the Mathematically Talking Parrot squeaks, "8 ones plus 5 ones equals 13 ones. Write the 13 in the thinking and trading area in the ones column." As Gandalf says this, Tinkerbell the Writing Gorilla writes "13" in the thinking and trading area in the ones column. Gandalf then squawks, "Trade 10 ones

for 1 ten. 13 ones is now the same as 1 ten and 3 ones. Write the 3 in the ones column and carry over the 1 ten and write it in the tens column." As Gandalf says this, Tinkerbell the Writing Gorilla writes a "3" in the ellipse at the bottom of the ones column to represent the 3 ones and a 1 in the small box at the top of the tens column to represent the 1 ten that is carried over to the tens column.

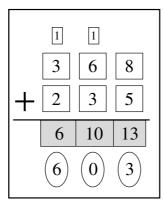
Exhibit 1.28



Doris now repeats this demonstration for the tens and hundreds column. When she finishes, she continues.

Gandalf the Mathematically Talking Parrot finally squeaks, "Three hundred sixty-eight plus two hundred thirty-five equals six hundred three," as Tinkerbell points to this in the wall's addition graphic.

Exhibit 1.29



Everything is quiet for a minute as if something more needs to be done. Habble starts jumping up and down while pointing at the marble wall with the two bands of musicians. Gandalf then adds, "If a band of 368 musicians joins with a band of 235 musicians, then there will be one band with 603 musicians!"

As soon as Gandalf the Mathematically Talking Parrot says that, the color of the room flashes pink and purple for two minutes as the numbers in the addition graphic burn away to nothing in flames and as the two marching bands march together to form a single band of 603 musicians and then explode into fireworks that sparkle with 603 tiny points of bright white light.

Tinkerbell and Gandalf feel very proud of themselves as they say magic words that turn themselves back into their normal selves. They worked together cooperatively, listening to and watching each other carefully as they coordinated their talk and writing.

Next, Habble and Bondo take their turns. Or rather, you students take their turns for them, so listen carefully to what happens and keep track of any mathematical things that you might need to use when you do the problem for Habble and Bondo.

Next, the marble wall begins to glow and on the upper half of it a baker appears carrying a muffin tin filled with one hundred delicious-looking cupcakes, each of which is decorated with precious diamonds that twinkle in the dim light of the cave. The cupcakes are arranged in the muffin tin in neat rows and columns with ten rows of cupcakes and ten cupcakes in each row. Following the first baker are three more bakers carrying muffin tins each holding one hundred cupcakes. Following them is a baker carrying a muffin tin with only five rows of ten cupcakes in it. And following him is a baker carrying a tray of six individual cupcakes. When all of these bakers and cupcakes are on the upper half of the wall, a second set of bakers carrying cupcakes decorated with blue sapphires walks onto the lower half of the wall. They carry two muffin tins of one hundred cupcakes each, one muffin tin with six rows of ten cupcakes, and four individual cupcakes.

When all of the bakers are lined up and standing still, the wall's addition graphic begins to glow violet, and with a screeching sound new numbers appear in it. And what do you think the numbers are?

Doris asks her students to meet in their groups and write the numbers on a scrap of paper and then raise their hands. When hands go up she checks to see that 456 and 264 are recorded. One group makes a mistake, and she asks a group sitting next to them to help them.

Doris has now demonstrated to her students how to do one problem, carefully demonstrating Gandalf's verbalizations and Tinkerbell's writing. She has also set up a second problem.

Doris tells her students that in their groups of two they must help the wizards by acting out the second problem given by Thoughtful Mountain as though they are the wizards. One member of each group must pretend to be the giant talking parrot, hold up the wood parrot when speaking, and be the verbalizer. The other group member must pretend to be the writing gorilla and use a chartreuse crayon to record numbers in one of the addition graphics on

their worksheet, numbers that describe what the gorilla says. Doris tells her students that there will be a third problem given by the mountain and that they will switch roles for this problem so that each person in their group will have a chance to be a talker and writer.

Doris has students choose roles and get the symbol of their role: a wood parrot or chartreuse crayon. Next, she has her students sing "Tinkerbell's Addition Song" and again reminds them of the meanings of some of its phrases. She also reminds them that they are to work cooperatively and what this means.

Doris's students start working the problem 456 + 264. Parrots are raised into the air as squeaky voices verbalize mathematics. Chartreuse crayons are carefully applied to worksheets. Students monitor each other's behavior and discuss what to do next. As before, Doris circulates among her students observing them and providing help as needed.

When the second problem is correctly completed, Doris asks students to switch roles as writer and talker and gives each group a third problem: 358 + 328. Over the years, Doris has embedded these problems in a variety of real-world situations: for example, Canadian geese flying in formation (a square of one hundred, a line of ten, and individual birds) or farmers carrying cartons of eggs (big square cartons of one hundred, long skinny cartons of ten, and individual eggs). The problems often relate to situations encountered in her classroom that the students recognize.

As before, when groups complete the third problem, Doris asks students to discuss in their small groups how the work they did relates to mathematics they might have learned elsewhere, and to record their conclusions. When small group discussions are complete, Doris holds a whole-class discussion during which students report their thoughts.

When the discussion is complete, Doris continues "The Wizard's Tale." As before, Thoughtful Mountain offers freedom or the granting of a wish. Bondo chooses that Thoughtful Mountain release Zunk. Then . . .

There is the sound of breaking stone, and Zunk falls out of the stone wall. Zunk is in the form of an evil witch, the form that the wizard who was annoyed by her playfulness gave her. Zunk takes one look at Gandalf, Tinkerbell, Habble, and Bondo and starts hissing magic curses. "Gandalf, gum, gam, gibble," she says and turns Gandalf into a big squirmy worm. "Habble, gum, gam, gibble," she says and turns Habble into a big squirmy worm. Tinkerbell quickly flies out of sight and hides. "Bondo, gum, gam, gibble!" yells Zunk and turns Bondo into a worm. Then she goes over to the worm Bondo and with great delight on her face, raises her foot to prepare to squash Bondo.

But Tinkerbell acts first. She gets ready to say some magic words [Doris signals and magic clapping occurs.], "Ding, bing, ping, swing Zunk" [more magic clapping]. Tinkerbell turns Zunk back into her puppy dog self. Then she turns Gandalf, Habble, and Bondo back into themselves.

Zunk immediately runs over to Bondo with her tail wagging and making whining sounds of delight upon seeing her old master. Bondo picks up Zunk and gives her a hug. Zunk starts licking Bondo in the face. But before anything else can happen, the mountain begins to rumble and boom.

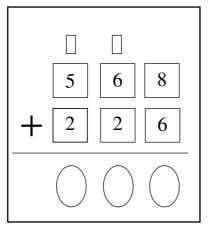
Doris now ends this episode of the story.

## DAY 5

This is the last installment of "The Wizard's Tale" and Doris wants to get her students prepared to do multidigit addition at a symbolic level without external aids. She also wants to extend addition beyond three digit numbers, connect multidigit addition with money, and check to make sure that her students know what to do when a column sums to 0.

The first part of Day 5 functions as a review of the previous day with students working in groups of two using a sheet of paper with four new transitional addition graphics on it. The purpose of the new transitional addition graphic is to wean students away from recording numbers in the thinking and trading area. The new graphics do not contain the thinking and trading area, and while solving problems it is necessary for the wizards to imagine the invisible thinking and trading area even though it is impossible to write anything in it. As on Day 4, Thoughtful Mountain announces that, "Two by two wizards go as mathematically talking parrot and writing gorilla!" It then presents the problem 568 + 226 using a display of jewels on its cavern's wall in which dollars (that turn into square arrays of one hundred pennies) are made out of opals and emeralds, dimes (that turn into lines of ten pennies) are made out of silver and rubies, and pennies are "intricately carved pieces of jade inlaid with diamonds." This is what the new transitional addition graphic looks like.

Exhibit 1.30



Day 5 then proceeds as did Day 4: Doris's students discuss how much money is on the wall and yell their answer in unison; the problem appears in a new addition graphic on the cave's wall; Tinkerbell and Gandalf magically light their stars and moon (with the help of magic clapping from Doris's class); Tinkerbell and Gandalf decide to be talking parrot and writing gorilla respectively; Doris's students sing "Tinkerbell's Addition Song," discuss how to do addition, and telepathically send their thoughts to the wizards; the wizards with the telepathic help of Doris's students describe to Zunk (the dog) how to do addition and then

comment that each time they have to find words to help them describe how to do addition it helps them better understand the process themselves; Tinkerbell and Gandalf magically turn themselves into talking parrot and writing gorilla (with the help of Doris's students' clapping); and Doris demonstrates (while speaking in a different tone of voice to represent each wizard) how to do the problem on the wall while "imagining" the existence of a thinking and trading area (which no longer exists in the new addition graphic).

When Doris finishes reviewing the previous day's work and relating addition of money to addition of numerical symbols, she asks her students to help the wizards solve two problems in their pairs, as they did the previous day. While solving the first money problem, 177 + 277, they pretend they are Habble and Bondo. When solving the second money problem, 389 + 151, they pretend they are Tinkerbell and Gandalf. During the problems they practice imagining the now invisible thinking and trading area in their minds while adding numbers in each column and making trades. While solving each problem, one student acts as the writer and uses the chartreuse crayon on one of the new addition graphics on the worksheet and the other acts as the speaker and holds a wood parrot while speaking. As before, students switch roles after each problem.

When students finish their problems and everything disappears from the cavern except for the magic stone pencil, which was left on its shelf on the wall, Doris continues the story.

Then Thoughtful Mountain gives three loud booms and says, "Tinkerbell and Gandalf, Mathematically Talking Giant Parrot and Writing Gorilla, on the wall add 345 and 456." The mountain then becomes quiet.

Gandalf says, "Just as I expected, now we have to do the addition without the help of any addition graphic. Remember those numbers: 345 and 456. Tinkerbell, do you want to be the parrot or the gorilla?"

Tinkerbell says, "I'll be the parrot and you can be the gorilla. But what do we do?"

"Yes," joined in Bondo and Habble, "what do we do when there is no addition graphic?"

"Easy," said Gandalf, "just pretend that the addition graphic is there and put numerals in its invisible boxes and ellipses! It's a cinch! Just look at the wall. Can't you see where the addition graphic is? . . . See it there? . . . There are the two little boxes up top, the six bigger boxes in the middle. . . . The line and the addition sign is under them, the thinking and trading area under that, and on the bottom are the three ellipses. And everything is arranged in nice neat columns so that as the song says, we can add

Column by column

Column by column.

"Just remember 'Tinkerbell's Addition Song' and imagine the addition graphic in your mind."

With the help of clapping from Doris's class, Tinkerbell and Gandalf now magically turn themselves into a talking parrot and writing gorilla so that Doris can demonstrate how to do this new problem without an addition graphic. In what follows she records the addition on her blackboard.

Tinkerbell then speaks up, "We are going to add 345 and 456, so we must first write down these numbers, one above the other with ones above ones, tens above tens, and hundreds above hundreds."

When this is said, Gandalf the Writing Gorilla [Doris] writes the problem on the wall with his magic pencil.

Then Tinkerbell the Mathematically Talking Giant Parrot says, "Draw a line below the numbers we are going to add that separates the problem from its answer and put an addition sign to the left of the lower number." Gandalf [Doris] does this, and the wizards are ready to begin adding.

#### Exhibit 1.31

Feathery Tinkerbell then says, "First we add the ones. 5 ones plus 6 ones equals 11 ones. Next the 11 ones are traded for 1 ten and 1 one in the invisible thinking and trading box. 1 ten and 1 one are now legal. So we write a 1 at the bottom of the ones column, move the ten over to the tens column where it belongs, and record it at the top of the column." While she says this, Gandalf [*Doris*] is at work writing on the wall of the cavern everything she says.

## Exhibit 1.32

Doris continues working the problem in this manner until it is complete, and this is what is on the wall.

## Exhibit 1.33

Next Habble and Bondo will have to take their turn, or rather you children will take their turn for them.

Doris has now both discussed and demonstrated how to do Thoughtful Mountain's latest problem with her students. She has both provided a cognitive map of the addition process and modeled the desired behaviors in such a way that her students can see how the writing takes place, hear how the mathematical thought processes are verbalized, and understand the correspondences between writing and verbalizations.

Doris gives her students the next two problems orally. They are 484 + 217 and 284 + 247. Her students work in pairs, as before, with one child acting as a writer and the other as a talker and switching roles between problems. This time, however, they write on a blank sheet of paper.

When her students finish adding, Doris continues the story.

Then Thoughtful Mountain gives three loud booms and says, "One last problem for you little wizards, 567 + 678. Each of you work it alone, compare your work, and tell me the answer." There is a sudden screeching sound, and out of the floor of the cavern grow four stone desks, one in front of each wizard. On each desk is a stone slate and a magic stone pencil. There is a sudden sound of trumpets from within each stone desk. And then the mountain becomes quiet.

Gandalf says, "I guess we each have to do the problem ourselves. Probably to prove to the mountain that we can each do addition. To the task wizards, and we will compare our answers when we are done! Remember the problem, 567 + 678. It is a tricky one, but I am sure that we can each do it. Just remember, if you start doing something, keep going in the same way until you finish."

Doris has now presented her students with a problem designed to help generalize the addition algorithm by extending addition beyond the hundreds place. Each student must now solve the problem individually, on a plain sheet of paper, speaking to himself or herself

as they write. When they finish, they are to check their written work, verbalizations, and answers with their previous partner and help their partner if necessary. Doris monitors her students' work.

(Laura McBride says that some of her third graders do not know what to do after adding in the hundreds column—even though they have previously worked with base ten block ones, tens, hundreds, and thousands—and that this stimulates wonderful class discussions.)

When groups finish their work, Doris's students telepathically send their thoughts to the wizards in Thoughtful Mountain (using the clapping process). First, they send the problem's answer (1,245) by yelling it in unison. Then, after sitting quietly and thinking about what they have done, they quietly send their thoughts about addition.

Then, as on previous days, Doris has small groups discuss and record how the work they did today relates to mathematics they might have learned elsewhere. This is followed by a whole-class discussion.

After the discussion, Doris continues the story.

As soon as the problem is complete and all of the wizards agree on its answer, the color of the room flashes from gold to silver to purple over and over again for several minutes as all of the desks, pencils, and slates in the cavern disappear in great fountains of sparkles. Then Thoughtful Mountain gives three loud booms and says, "Very good, little wizards. You can now either be released from me to go in peace or have any wish that is in my power to grant."

Gandalf replies, "Release us from your cavern to go in peace!"

As soon as he says that, Thoughtful Mountain starts to make the sounds of a volcano beginning to erupt. All of the wizards move together in a fearful cluster, and Zunk jumps into Bondo's arms. Below their feet, the floor of the cavern turns to molten lava that burbles like a caldron of liquid red stone. For some magical reason, however, it does not burn their feet or roast them. Then suddenly the roof of the cavern opens up in the shape of a volcano's crater and blue sky shows up above. Gradually the molten lava below their feet bubbles up through the core of the mountain and the wizards are carried to the top of the volcano's rim. Then, all of a sudden, the wizards are shot up into the sky like cannonballs shot from cannons as Thoughtful Mountain erupts and spews smoke, lava, and them into the sky.

As soon as they are thrown into the sky, Gandalf and Tinkerbell turn themselves into giant eagles with the magic words [Doris signals and magic clapping occurs.] "Eagle, agle, oggle, ungle" [more magic clapping]. Gandalf flies under Habble, who lands on his back. Tinkerbell swoops under Bondo, who is still holding Zunk, and the two of them land on her back. Then the two wizardly eagles with their cargoes fly away from Thoughtful Mountain as its volcanic eruption subsides and it closes its volcanic crater so that it looks like it has never been a volcano.

Gandalf squawks to Tinkerbell in an eagle's voice, "Follow me to my home. It is closer than yours!" And the two eagles fly off to Gandalf's home.

When they reach Gandalf's house, the eagles put down their cargoes and turn themselves back into their normal forms of Gandalf and Tinkerbell. Zunk barks loudly to let everyone know that she is hungry. All of the others agree. They go into Gandalf's dining room, and Gandalf creates a wonderful dinner for them by turning dust into food. There is plenty of food because there is lots of dust in Gandalf's house. And what a wonderful meal it is, with heaps of meat and vegetables, but the best part is the desserts. There are three wonderful ice cream desserts, each of which has fireworks exploding over it and music coming from within it.

When the meal is finished, the three wizards and the puppy dog leave Gandalf's house, each to return to their own homes.

And this is where the story ends.

This is not where Doris's instruction on multidigit addition ends, however. Following the story Doris's students play numerous homemade academic skill development games to maintain and further develop their mathematical skills and understanding of addition. Doris makes the games out of such things as egg cartons, wood cubes, poster board, tongue depressors, and printer's cards. Doris also introduces her class to computer games that require multidigit addition.

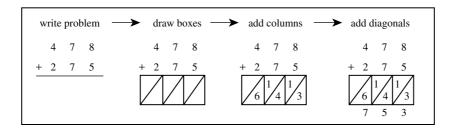
Later in the year, Gandalf sends Doris's class letters addressed to Tinkerbell that describe different methods of doing addition that he discovers while traveling around the world. These alternative algorithms introduce Doris's students to ways in which other cultures do addition and help students clarify and deepen their understanding of the way in which they were taught the operation. A copy of one letter follows.

#### Exhibit 1.34

Dear Tinkerbell,

The Museum of Ancient Wizardry has asked me to search throughout the entire world for outstanding portraits of wizards. I am traveling everywhere looking for pictures. I am now in Italy.

While here I discovered an interesting method of doing addition. I was told that it was invented over 400 years ago in Italy. This is how it works.



Can you figure out how this addition method works?

Do you think that this old Italian method of addition always gives the same answer to addition problems as the addition method that we used in Thoughtful Mountain?

I think that this method is similar to ours in some ways, and I also think that it is different in some ways. Will you explore how this method is similar to and different from the method we used?

My magical regards, Gandalf