THE DOUBLING GAME

From Thomas Malthus to Bernie Madoff

To understand how our world has become stuck in a seemingly endless series of economic crises, we first look at a trend that is increasingly causing distress to the planet, is most certainly man-made, and is no longer sustainable given the nonfrontier global world in which we live. In fact, the early seeds of capitalism in the British Industrial Revolution followed by the frontier thinking of the expansionist United States (as witnessed in the opportunist battle cry "go west, young man" and in the extraordinary increase of industrial output during the two world wars) has allowed this trend to embed in our modern economic thinking and to become the leitmotif of an entire civilization, the implication of which is serious for all and has at its core our own greed.

I am referring to the doubling game, which was first told as a tale in ancient India and has since fueled the ambitions of many a fortune seeker, from tulip traders to property speculators to stock-market dealers. Here, we look at its origins in ancient India and how it played a role in a few modern examples, such as the 1929 stock market crash, the 2009 credit crisis, and the billion-dollar pyramid scams of New Yorker Bernie Madoff and Texan Allen Stanford. All hold the belief that continuous growth can last forever, endlessly unchecked.

Many examples of doubling appear in our lives. Backgammon has a doubling cube to a maximum of 64 ($2 \times 2 \times 2 \times 2 \times 2 \times 2$ or 2^6 for a six-sided die). James Bond's favorite game, baccarat, allows a doubling of the bet ad infinitum. European paper sizes[1] are made in exact doubling ratios, where the standard-size writing paper, A4, equals one half the next size, A3, and so on. The television quiz show *Who Wants to Be a*

Millionaire has bits of geometric-like doubling in its cash prizes ($100, $200, $500, $1,000, . . .), as do population sizes over time (3.3 billion in 1965, 6.5 billion in 2006), cells doubling by mitosis, and even infectious diseases. Even the tribbles of *Star Trek* fame grew exponentially, with just one multiplying to 1,771,561 in 3 days via a tenfold growth every 12 hours, according to Spock's calculation.

The semiconductor industry has generally followed "Moore's Law"—first proposed in 1964 by Gordon Moore (1965), cofounder of Intel—where the number of transistors on a chip (or chip density) doubles every 2 years. The number of transistors on a standard computer processor now stands at more than 4 billion, or 32 doublings (2^{32} = 4,294,967,296). Correspondingly, the transistor size has shrunk and will eventually reach atomic dimensions.

But can such doubling be sustained? How long before there are no more sides to a die, the bank says no, or atomic dimensions reach their limits? How long before a piece of paper can no longer be folded? Is there a limit to the doubling game?

A series of coin tosses or roulette spins (or any two-outcome or binomial test) is the mathematical basis for doubling logic, where one can easily count the sample space (number of total outcomes) versus all possible outcomes. For example, we'll see later how to calculate the probability of getting any mixture of heads and tails on a series of coin tosses, the chances of landing on red or black in a number of roulette spins, or the possibility of correctly predicting an up/down stock market index over a number of weeks. For now, each flip, spin, or pick is simply a doubling of outcomes (e.g., either heads or tails, red or black, win or lose).

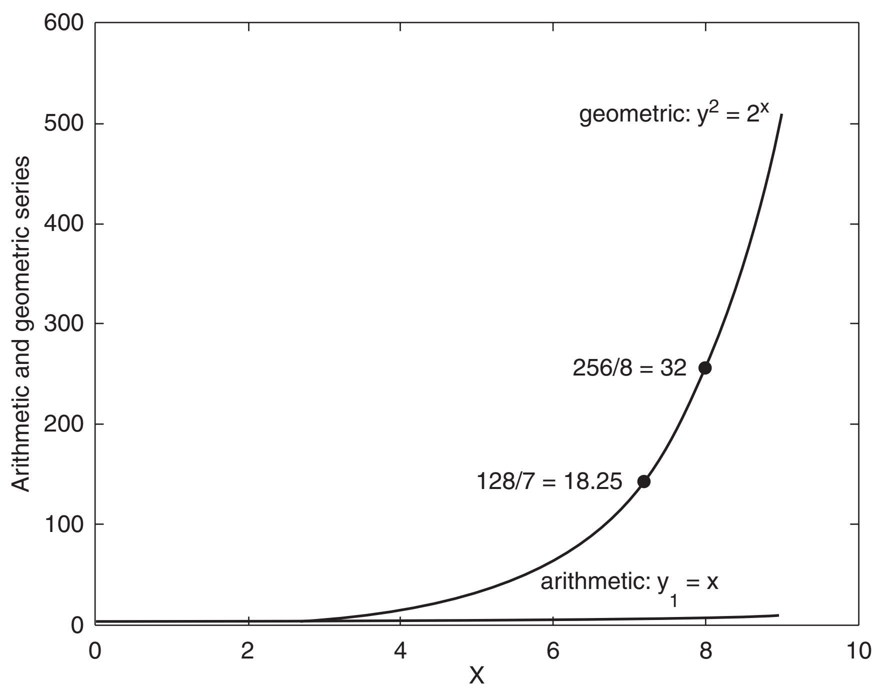
In the same way, a doubling or geometric progression can inform us about a pyramid scam, where she told two friends, who told two friends, who told two friends, and so on, and before you can say "Bob's your uncle" or "Bernie Madoff is a crook," there are no more friends, thanks to the exponential growth of doubling: $2 \times 2 \times 2 \times 2$ . . . . Some also see the doubling game in government pensions, which continually pull in young workers to pay older retirees,[2] or in continuous bond issues to pay off debt in a seemingly perpetual payout machine that postpones a future reckoning.

Thomas Malthus (1798), the English mathematician and clergyman, wrote about such progressions in his *Essay on the Principle of Population*, stating that the world's population would double (geometric) while the food supply would increase only linearly (arithmetic), spelling future disaster and eventual world famine. "A slight acquaintance with numbers will show the immensity of the first power in comparison of the second," wrote Malthus (p. 4), basing his theory of the inability to maintain subsistence for a constantly growing population in part on the rapid growth of the Irish population due to the success of the potato plant, prior to its subsequent failure in the 1840s.[3]

The effect of the two functions is easy to see when plotted (see Figure 1.1). The *geometric* function is written as $y = 2^x$ (also referred to as an exponential function because of the raised exponent x). The *arithmetic* function is a straight line and is written as $y = Ax$, where the constant A is the slope, indicating how fast the line rises for every x (in the figure, A = 1). In Malthus's description, $y = 2^x$ is the world population and $y = x$ is the food supply, where a simple comparison of the two ($2^x/x$) shows how quickly the geometric "doubling" (2, 4, 8, 16, 32, . . .) swamps the arithmetic "stepping" (1, 2, 3, 4, 5, . . .).[4]

As can be seen, after three steps ($x = 3$) there is almost no difference (8/3), but soon the geometric progression outpaces the arithmetic

Figure 1.1 Arithmetic ($y_1 = x$) and geometric progression ($y_2 = 2^x$)



progression, and all the more as they both increase. After seven steps ($x = 7$), the ratio of the geometric to the arithmetic progression is almost 18 times greater (128/7), while after eight steps the ratio is already 32 times greater (256/8).

Here, a fable helps highlight the mathematics. As the story goes, a clever courtier gave an Indian king a beautiful chess set, and the grateful king asked what the courtier would like in return. According to the tale, the courtier asked for a grain of rice doubled every day for each square on the chessboard. Thinking the request simple, the king agreed—laughing, it is said—and on the first day, the courtier was given a single grain of rice. The next day, he was given two grains of rice, and the next four. After 1 week, the courtier received 64 grains of rice ($1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$) for a total of 255 ($2^7 - 1$). The king, it is said, was still laughing.

Continuing with their agreement, after 2 weeks the courtier received 2^{13} or 8,192 grains, although by then the king didn't seem to be laughing as much. By the end of week 3, the courtier received 2^{20} or 1,048,576 grains, and the king wasn't laughing at all. In fact, the king was downright annoyed, as his rice supply was noticeably diminishing and they were still only at square 21. Soon, after most of the rice had been delivered to the smiling courtier, the king was beside himself.

Alas, by the end of week 4, the jig was up, although it is said that the king got the last laugh—he had the all-too-clever courtier executed. The story has also been told with pieces of gold instead of rice, for which the courtier was executed far sooner.

To help clarify the impossibility of sustaining such geometric progressions, the payout for each square on the king's chessboard is shown in Figure 1.2, where it can be seen that had there been enough rice to continue until the last square and had the king continued to pay, the courtier would have received 2^{63} or 9,223,372,036,854,775,808 grains of rice for a total of $2^{64} - 1$ or 18,446,744,073,709,551,615 (more than 18 trillion million). To illustrate how large 18 trillion million is, a 500-gram box of Uncle Ben's rice (about four servings) has about 32,000 grains, which would require every person in the world to eat two boxes every day for more than 1,000 years to equal 18 trillion million—numbers that boggle the mind, and readily show the impossibility of endless doubling.

Of course, the rate at which doubling occurs is also a concern. It is one thing to hand over a bag or two of rice but quite another to start delivering loads in ever-increasing sizes—first wheelbarrows, then trucks, then container ships. At the start, the means may be simple, but

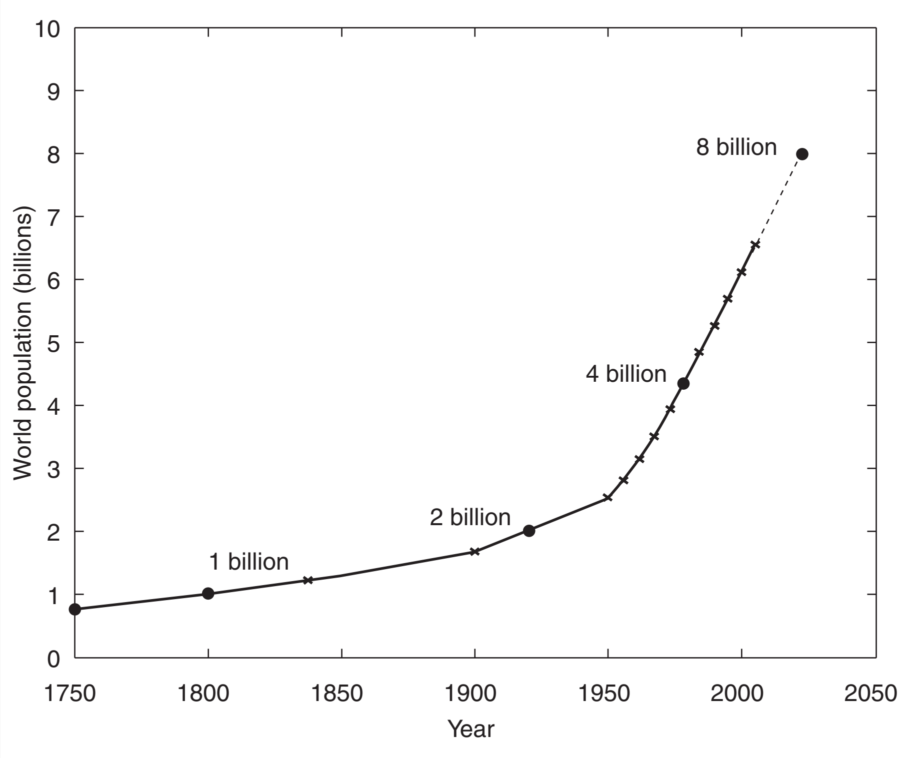
Figure 1.2 Chessboard doubling (2^{64} = 18,446,744,073,709,551,616)

Square 1 Day 1 1	Square 2 Day 2 2	4	8	16	32	Square 7 End week 1 64	128
256	512	1024	2048	4096	Square 14 End week 2 8,192	16,384	32,768
65,536	131,072	262,144	524,288	Square 21 End week 3 1,048,576	2,097,152	4,194,304	8,388,608
16,777,216	33,554,432	67,108,864	Square 28 End courtier 134,217,728	268,435,456	536,870,912	1, 073,741,824	2, 147,483,648
4, 294,967,296	8, 589,934,592	17, 179,869,184	34, 359,738,368	68, 719,476,736	137, 438,953,472	274, 877,906,944	549, 755,813,888
1,099, 511,627,776	2,199, 023,255,552	4,398 046,511,104	8,796, 093,022,208	17,592, 186,044,416	35,184, 372,088,832	70,368, 744,177,664	140,737, 488,355,328
281,474, 976,710,656	562,949, 953,421,312	1,125,899, 906,842,624	2,251,799, 813,685,248	4,503,599, 627,370,496	9,007,199, 254,740,992	18,014,398, 509,481,984	36,028,797, 018,963,968
72,057,594, 037,927,936	144,115,188, 075,855,872	288,230,376, 151,711,744	576,460,752, 303,423,488	1, 152,921,504, 606,846,976	2, 305,843,009, 213,693,952	4, 611,686,018, 427,387,904	9, 223,372,036, 854,775,808

eventually the infrastructure is unable to keep up. Simply put, such growth cannot continue unabated.

Figure 1.3 shows the doubling of the world population in time (United Nations, 2011) and clearly shows a trend of exponential growth (circles mark the doubling periods). What is most revealing here is that the doubling period has been shortening from one double to the next—300 years from 500 million to 1 billion (1500 to 1800), 125 years from 1 billion to 2 billion (1800 to 1925), and 50 years from 2 billion to 4 billion (1925 to 1975), until recently leveling off (54 years from 4 billion to a projected 8 billion, 1975 to 2029). In the process, the population has been doubling faster and, as such, consuming resources faster, with significant consequences to the environment.

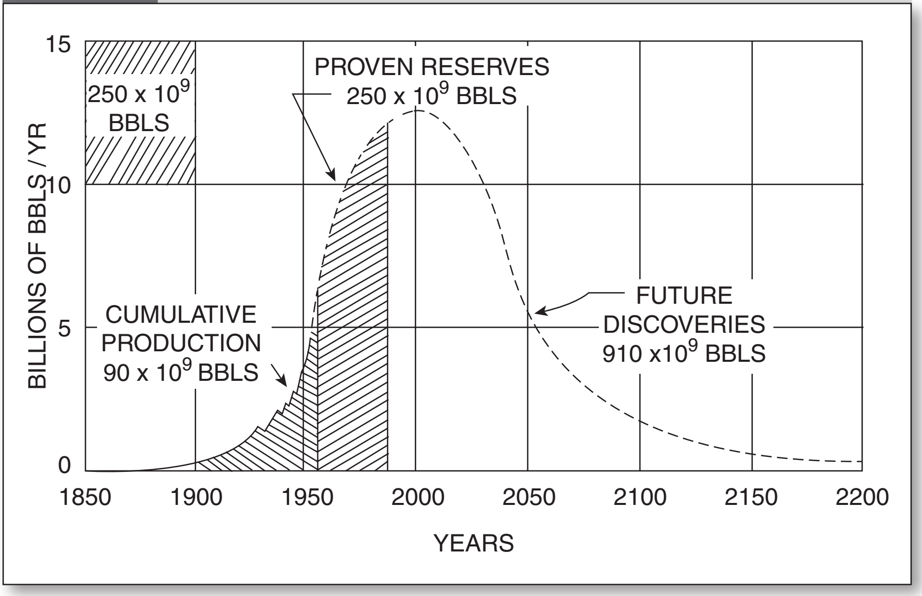
Figure 1.3 World population versus time



Indeed, economic productivity and national production have grown enormously since the beginning of the Industrial Revolution in the early 19th century, which has prompted new questions with regard to the earth's resources: Are there limits to continued growth and, if so, when will it end? Will we run out of coal, oil, gas, and other natural resources? What's more, will we see the end coming?

Oil output is a good example of a doubling game, showing huge increases throughout the 20th century, doubling in the United States every 8.7 years from 1880 to 1930 (Hubbert, 1956, p. 7), and prompting questions about its continued sustainability. The concept of peak oil (defined as half the available oil in the ground to be extracted) originated with the American geophysicist M. King Hubbert (1956), who predicted that the United States would reach maximum production (*peak* oil) in the 1970s, which it did (p. 24). Figure 1.4 shows his further prediction for world peak oil in 2000.

Figure 1.4 Hubbert's 1956 figure showing world oil production peaking in 2000 (p. 20)



SOURCE: Marion King Hubbert Collection, Box 85, Folder 4, American Heritage Center, University of Wyoming.

Globally, peak oil is still debated, although annual discoveries of new oil have declined since the 1960s (Hobbs, 2010, p. 33) and may be declining by as much as 9% per year (Jackson, 2009, p. 32). Some estimate that oil supplies have already peaked (Hobbs, 2010, p. 36; Hubbert, 1956, p. 20), others that they soon will (Fleming, 2011a). The International Energy Agency predicts "a significant potential gap between supply and demand by 2015" (Moyo, 2011, p. 165). Note, however, that what has already been extracted is the easier half—the upside—and that the downward half will be much more difficult, not to mention scarcity- or security-based price increases as we approach "end" oil.[5]

In his 1973 book, *Small Is Beautiful: A Study of Economics as if People Mattered*, the British economist E. F. Schumacher continued in the tradition of Malthus, warning us that we are using up fossil fuels at an alarming rate. He suggested that fossil fuels, what he called "natural capital," should be treated as capital and not as income items, thus necessitating our concern for their conservation. Natural capital is not limitless, doled out forever like collecting $200 every time one passes "Go" in Monopoly. Fossil fuels cannot be replenished, and, thus, the question is not *will* we run out of resources—which of course we will—but *when* and what damage will be done in the process? Furthermore, will we ultimately destroy the world? As Schumacher (1993) noted,

> We are not in the least concerned with conservation; we are maximising, instead of minimising, the current rates of use; and, far from being interested in studying the possibilities of alternative methods of production and patterns of living—so as to get off the collision course on which we are moving with ever-increasing speed—we happily talk of unlimited progress along the beaten track, of "education for leisure" in the rich countries, and of "the transfer of technology" to the poor countries. (p. 4)

In two tables from his book, Schumacher (1993) showed population, fuel consumption (in million tons coal equivalent), and fuel consumption per head for "rich" and "poor" countries[6] in 1966 and a projected year, 2000. Malthus's geometric doubling readily appears in these tables: The world's population doubles (6,909/3,384), while the fuel consumption more than quadruples (23,156/5,509; pp. 13–14)—a striking result, borne out by recent figures showing the doubling of population growth and the quadrupling of fuel consumption since 1966. Furthermore, his data shows that 23% of the world was consuming 67% of the fuel in 2000, but that the "poor" nations of today are consuming more than the "rich" nations of only 34 years ago (7,568 million tons vs. 4,788 million tons coal equivalent), and in a greater percentage (1/7 or 1.43/9.64 in 2000 compared with 1/14 or 0.32/4.52 in 1966; pp. 13–14). What's more, China and India (considered poor in his survey) are now growing in Western ways,[7] and, thus, one must ask, is there enough fuel left to sustain first-world growth for another 34 years?

Another indication of energy use is the level of carbon emissions, which is also an area of increasing concern with regard to global warming. The American Museum of Natural History's Center for Biodiversity and Conservation stated that "people in the United States and Canada account for approximately 5.3% of the global population, yet they produce about 26% of global CO_2 emissions" ("Per Capita Resource Consumption," 2011), and, thus, if the rest of the world were to consume at the same rate per capita, the earth would need almost 5 times its current resources. The Center further stated that "Canadians, U.S. citizens, and European Union members are generally considered to have comparable standards of living, yet Europeans on average use 47.2% as many resources per person (in oil energy equivalent units) as their North American counterparts" ("Per Capita Resource Consumption," 2011). The International Energy Agency also predicted that carbon emissions will almost double in the next 20 years, three-quarters of which will come from China, India, and the Middle East (Moyo, 2011, p. 167). Other figures show that China now emits twice as much CO_2 from coal as all of Europe does (Hobbs, 2010, p. 105). What's more, the demand for resources is increasing.

But can current industrial resources sustain such growth? Recent growth has been spectacular, spurring on our over-consuming ways. Jackson (2009) noted about current supplies, "If the whole world consumed resources at only half the rate the U.S. does, for example, copper, tin, silver, chromium, zinc, and a number of other 'strategic metals' would be depleted in less than four decades" (p. 10). Furthermore, will there be enough food to sustain an ever-increasing population? If the population continues to grow at the current rate, a third more food will be required in 15 years (Zakaria, 2009b, p. 30) and 50% more by 2050 (Jackson, 2009, p. xvii).

According to Goldman Sachs, "the combined GDP of the four BRIC[8] economies—Brazil, Russia, India, China—could overtake the combined GDP of the G7 countries by 2035. These days, they say it could happen by 2027" (Zakaria, 2009b, p. xxii). To put the numbers in perspective, based on population alone, an increase in use of only 5% in Brazil, Russia, India, and China is equivalent to a 50% increase in the United States. If those in the West are already starting to see limits in available resources, imagine how much worse it can get given those figures.

Mason (2009) also recited the numbers:

> In 2007 global GDP growth was 5%—well above its historic trend for the fourth year in a row. Growth in the developing world averaged 8%; and in Asia it was 10%. Across the G7 countries it was 2.6%—slightly below the average for the 1990s. (p. 157)

To be sure, the growth has been staggering and without precedent. But as Jackson (2009) noted, "This extraordinary ramping of global economic activity . . . [is] totally at odds with our scientific knowledge of the finite resource base and the fragile ecology on which we depend for survival" (p. 13).

What's more, with regard to the effects such growth has on our economies, American economics professor Hyman Minsky noted that "the normal functioning of our economy leads to financial trauma and crises, inflation, currency depreciations, unemployment and poverty in the midst of what could be virtually universal affluence—in short . . . financially complex capitalism is inherently flawed" (Mason, 2009, p. 154).

And yet we are told that growth is the only model, unchecked until a correction occurs—often disastrously, as in the 2009 credit crunch and resultant worldwide recession,[9] which in some countries was in fact a depression. As noted by Morgan Stanley economist Stephen Roach, "A voracious appetite for economic growth lies at the heart of the boom that has now gone bust" (Mason, 2009, p. 157).

Of course, we should always examine the goals of industry and consider the effects such growth has on social patterns and sustainable living. John Stuart Mill reminded us that the ultimate aim of economics is toward "a stationary state of capital and wealth" and that the ever-increasing demands of an ever-expanding economy are simply incompatible with the ever-dwindling resources of a finite supply (Jackson, 2009, p. 122). As Jackson stated, "Economics—and macro-economics in particular—is ecologically illiterate" (p. 123).

Dwindling resources and increasing worldwide consumption have put us on a collision course between what we have and what we use, and while the West imports more and more oil, it is exporting more and more of its over-consuming ways—not to mention the increased waste and pollution from fossil fuel emissions and increased contributions to greenhouse gases and global warming. For a society based on a previously abundant supply of cheap oil, new strategies are needed—such as renewable energy and reduced use—and are imperative if we hope to continue our modern ways. The simplest mathematics of arithmetic and geometric progressions explain as much.

We seem to grow without any concern for sustainability, but a world that cannot reduce its over-consumptive ways may itself be consumed. A world that cannot reduce its waste may itself soon become waste. Just as in any failed doubling game.

Notes

1 **European paper sizes**: A4 (210 × 297 mm) = ½A3 = ½A2 = ½A1 = ½A0. An A0 poster (841 mm × 1189 mm) = 16A4.

2 **The ultimate Ponzi scheme**: Moyo (2011) doesn't pull any punches, stating, "Forget Bernie Madoff, forget Allan Stanford, the biggest Ponzi scheme has got to be the looming car crash that is Western pension funds. And like any well-run Ponzi game, its results will be devastating" (p. 79). She adds that 2008 American pension costs were $2.2 trillion or 15% of GDP, whereas in the United Kingdom they were an even more staggering $1.3 trillion or 64% of GDP (p. 79).

3 **Malthus and growth**: Zakaria (2009b) noted that Malthus's essay "is remembered today for its erroneous pessimism, but, in fact, many of Malthus' insights were highly intelligent" (p. 57). He further noted that "Malthus was wrong about Europe. His analysis, however, well described Asia and Europe" (p. 57).

4 **Malthus's influence**: Both early naturalists Alfred Wallace and Charles Darwin based the origin of their ideas about natural selection, where species must compete to survive, on Malthus's relation $2^x/x$. Herbert Spencer later applied the catch phrase "survival of the fittest."

5 **The end of oil?**: Hobbs (2010) noted that the estimated amount of oil remaining at the peak was 1,000 billion barrels, which amounts to less than 40 years left based on a current use of 35 billion barrels per year, and even less if demand from China and elsewhere increases (p. 36). Maass (2009) noted that "the advent of peak oil is yet another incentive to cut our dependency, because in the years ahead the price will only rise—skyrocket, really—if we fail to arrest our desires for it" (p. 223).

6 **Schumacher's rich and poor**: Schumacher (1993) defined a "rich" country as one with an average fuel consumption of more than one metric ton of coal equivalent (p. 13). He used United Nations figures throughout.

7 **Chinese and Indian energy consumption**: According to the United States Energy Information Administration (2011), world marketed energy consumption is expected to increase by 53% from 2008 to 2035, growing the most (117%) in non-OECD Asia (led by China and India). China's oil consumption increased by 6% to 8.3 million barrels per day (mbpd) (2008 to 2009) and its installed electric capacity more than 10% to about 800 GW (2007 to 2008; http://www.eia.doe.gov/EMEU/cabs/China/pdf.pdf). India's oil consumption is expected to increase by 3.2% to 3.1 mbpd in 2011 and its installed electric capacity by almost 50% to 232 GW (2007 to 2012; http://www.eia.doe.gov/EMEU/cabs/India/pdf.pdf). Note that almost 400 million people in India still have no access to electricity. The rank of the top seven oil consumers is USA (24%), China (8.7%), Japan (5.8%), Russia (3.2%), India (3.2%), Brazil (3.1%), and Germany (2.8%), totaling more than half the estimated 87 million barrels used per day (Hobbs, 2010, pp. 49–50).

8 **BRIC nations**: The BRIC nations (Brazil, Russia, India, and China) are a loose group of countries with similarly growing economic power. Recently, South Africa was added, giving the five-nation group the moniker BRICS. The nations all have large overall economies, but relatively poorer per capita incomes.

9 **Recession and depression**: A recession has been defined as a fall in GDP for two successive quarters and a depression as a fall in GDP for more than 3 years or a one-time drop of 10%. Neither is conclusively defined.